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Optimal Fiscal Policy in a Small Open Economy with Incomplete Markets and Interest Rate Shocks[1]

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Abstract

This paper studies optimal fiscal policy in a small open economy model under incomplete financial markets, where interest rates, government spending and productivity are stochastic and taxes are distortionary. The contributions of the paper are twofold. First, I solve the Ramsey problem and characterize the properties of the optimal fiscal policy. Second, I show that the optimal fiscal policy consists in smoothing tax distortions over time. The income tax rate and specially the public debt are very persistent irrespective of the degree of autocorrelation of the assumed processes for the shocks generating aggregate fluctuations. The government finances an increase in government spending or a decrease in the tax base partly by increasing debt and partly by increasing the tax rate. This reflects the government's desire to smooth the cost of raising taxes over time.
Keywords: optimal fiscal policy, taxation, incomplete markets, debt policy.
JEL Classification: E60, F34, F41, H21

Resumen

Este artículo estudia la política fiscal óptima en un modelo de equilibrio general para una economía pequeña y abierta con mercados financieros incompletos, donde la tasa de interés, el gasto público y la productividad son variables estocásticas y los impuestos son distorsivos. Las principales contribuciones del presente artículo son dos. Primero, se resuelve el problema de Ramsey y se caracterizan cuantitativamente las propiedades de la política fiscal óptima. Segundo, se muestra que la política fiscal óptima consiste en suavizar las distorsiones impositivas a través del tiempo. La tasa impositiva, y especialmente la deuda pública, son muy persistentes independientemente del grado de autocorrelación de los choques que generan las fluctuaciones en la economía. El gobierno financia un incremento en el gasto público o una caída en la base impositiva en parte mediante un incremento en la deuda pública y en parte mediante un incremento en la tasa impositiva, lo cual refleja el deseo del gobierno de suavizar a través del tiempo el costo de incrementar la tasa impositiva.
Palabras Clave: política fiscal óptima, impuestos, mercados incompletos, política de deuda.

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1 Introduction

In this paper, I analyze the properties of optimal fiscal policy in a small open economy where interest rates, government spending and productivity are stochastic, taxes are distortionary and markets are incomplete. This paper provides a new framework to analyze optimal fiscal policy in small open emerging economies. The present paper also contributes to the literature on optimal fiscal policy by studying the case of incomplete markets in a small open economy under uncertainty and distortionary taxation.

Small open emerging economies typically face frequent and large fluctuations in interest rates, productivity and government spending. These economies differ from small open developed economies in several aspects: For example, output and consumption volatilities are higher in emerging economies than in developed economies. Also, as pointed out by Neumeyer and Perri (2005), interest rates are countercyclical in these economies, while in developed economies interest rates are acyclical.

In particular, it is important to answer the following questions: What are the properties of optimal fiscal policy in a small open emerging economy under incomplete markets and distortionary taxation? How should the tax rate and the level of public debt adjust to an innovation in government spending or productivity in a small open economy? How should the tax rate and the level of public debt adjust to an innovation in the interest rate in a small open economy? In this paper, I analyze these questions using a stochastic dynamic general equilibrium model of a small open economy with incomplete markets.

I consider it is important to answer these questions, because, in my view, a model with incomplete markets captures better the financial environment faced by emerging economies than the complete markets models, because even though emerging economies may have perfect access to international financial markets, they can not borrow contingent on the state of nature as the complete markets models assume. Developed countries have access to a richer menu of financial assets than developing countries, and as shown by Angeletos (2002) and Buera and Nicolini (2002) governments can use the maturity structure of non-contingent public debt to replicate the complete markets optimal allocation as long as they have access to a sufficiently rich maturity structure.

Therefore, in this paper I solve the optimal fiscal policy problem for a small open economy under incomplete markets and distortionary taxation. In the case of complete markets, any tax schedule can be implemented as long as it satisfies the economy's resource constraints and as long as it can be sustained by a competitive equilibrium. When markets are incomplete, only a subset of the complete markets policies is available to the government. This introduces additional constraints on the set of competitive equilibrium allocations that the government can choose from, which makes the problem computationally more difficult.

In the model economy there are three agents, households, firms and a government. Households value leisure and consumption. Firms produce final goods using labor as the only input in production. In addition, firms have to pay for part of the wage bill before production takes place creating a need for working capital as in Neumeyer and Perri (2005)[3]. The government finances an exogenous and stochastic sequence of unproductive public consumption by issuing debt and by levying income taxes. The only taxes available to the government are proportional income taxes, which distort the consumption-leisure margin. All the agents in the economy have access to international financial markets, where they can borrow or lend to foreigners. Financial markets are incomplete because agents can only buy and sell one-period non-contingent real bonds. Moreover, I assume that agents can commit to repay their debt. The model economy is subject to three types of shocks: productivity shocks, interest rate shocks and government spending shocks.

I follow the Ramsey approach in characterizing the optimal fiscal policy. In this approach the Ramsey planner chooses an allocation that maximizes the household's utility subject to the condition that this allocation be implementable as a competitive equilibrium. In addition, I assume that the Ramsey planner commits to the announced policies.

The contributions of the paper are twofold. First, I solve the Ramsey problem for a small open economy with incomplete markets and stochastic interest rates, government spending and productivity. When markets are incomplete, as I have already mentioned, only a subset of the complete markets policies is available to the planner. This introduces into the Ramsey problem additional implementability constraints that arise from the requirement that the debt be risk-free. Since conditional expectations of future variables appear in these implementability constraints, the Ramsey problem is not recursive. Nevertheless, I show it is possible to recover a recursive formulation using the recursive contracts approach of Marcet and Marimon (1998). After setting up the Ramsey problem recursively, I compute the optimal policy by solving a log-linear approximation to the Ramsey planner's optimality conditions.

Second, I find that the optimal fiscal policy consists in smoothing tax distortions over time.
The income tax rate, and specially the public debt are very persistent irrespective of the degree of autocorrelation of the assumed processes for the shocks generating aggregate fluctuations. This reflects the planner's desire to smooth the cost of raising taxes over time.

The Ramsey planner finances an increase in government spending or a decrease in the tax base partly by increasing debt and partly by increasing the tax rate. In order to avoid a large distortion at the time of the shock, the planner smoothes the tax increase over time. As a consequence, the stock of public debt displays a persistent increase. Debt plays in this model an important role as a shock absorber. After a positive innovation in the interest rate or in government spending, or after negative innovation in productivity, the level of public debt and the primary deficit increase. Government debt responds on impact less than the other variables, but it accumulates over time, and displays the most persistent impulse response function. The responses of debt and the primary deficit have the same sign in the first periods. However, the response of the primary deficit changes sign after a few periods,

[3] Neumeyer and Perri (2005) introduced the working capital assumption in order to generate a transmission mechanism by which interest rates affect output.

because a higher debt interest will have to be serviced in the future in response to an increase in debt today.

The paper proceeds as follows. Section 2 briefly discusses the literature on optimal fiscal policy. Section 3 presents the economic environment and the theoretical model. Section 4 presents the Ramsey problem, and develops a recursive representation. Section 5 describes the calibration of the model and analyzes the quantitative results to illustrate the dynamic properties of the optimal fiscal policy. Section 6 concludes.

2. Related Literature

This paper is related to several studies about optimal fiscal policy. An extensive literature on optimal fiscal policy has emerged since the seminal work of Lucas and Stokey (1983). Most of the existing work, however has limited attention to closed economy environments. This paper instead studies optimal fiscal policy in a small open economy with incomplete markets. The two key elements that distinguish my analysis from the pertinent literature are the following: First, I consider a small open economy where the agents can only buy or sell risk-free debt. Second, I introduce stochastic interest rates to analyze how interest rates shocks affect the properties of the optimal fiscal policy.

In a closed economy environment with complete markets, Lucas and Stokey (1983) used the Ramsey approach of optimal taxation to study the properties of optimal fiscal policy. They found that it is optimal to respond to fiscal shocks by appropriately altering the state-contingent return on government debt and keeping the tax rate roughly constant, so state-contingent debt serves as an instrument to smooth tax distortions over time and states of nature. They also show that tax rates and debt inherit the serial correlation structure of the underlying shocks. Chari, Christiano and Kehoe (1994) analyzed the quantitative features of optimal fiscal policy in a standard real business cycle model with complete markets as in Lucas and Stokey (1983). They showed that another way to keep tax rates stable over the business cycle is to have non-state contingent debt with taxes on interest income that vary with the shocks, in this case state-contingent taxes on interest income should be used to provide insurance against adverse shocks. They found that in calibrated models to the U.S., the standard deviation of optimal income taxes is close to zero while taxes on interest income are highly volatile and serially uncorrelated.

Aiyagari et al (2002) restricted the government to issue only one-period non-contingent debt[4]. They showed that optimal fiscal policy under this environment imposes a near random walk behavior on taxes and debt irrespective of the degree of autocorrelation of the underlying shocks. They also found that the level of debt permanently increases after a fiscal shock, and that the response of the tax rate is a weighted average of a random walk and a

[4] Aiyagari et al (2002) modify Lucas and Stokey's economy to restrict the government to issue only one-period real non-contingent debt.

serially uncorrelated process. Their results affirm partially the random walk hypothesis of Barro (1979)[5].

Angeletos (2002), and Buera and Nicolini (2002) considered governments restricted to trading non-contingent real debt of different maturities. They showed that governments could use the maturity structure of non-contingent public debt to replicate the complete markets optimal allocation. However, Buera and Nicolini showed that the government might need to take extremely large long and short positions in debt of different maturities. Since all of the above papers study the properties of optimal fiscal policy in closed economies, they do not consider interest rate shocks.

In an open economy setting, Schmitt-Grohe and Uribe (2003b) study the properties of optimal policy in a monetary small open economy in which agents have access to complete international asset markets. They show that in a small open economy under complete markets optimal tax rates do not change in response to government spending shocks while in a closed economy they do. Riascos and Vegh (2004) consider an environment in which government spending is determined endogenously. They show that when markets are complete, the correlation between government consumption and output is zero. However, if markets are incomplete the correlation between government consumption and output is large and positive.

In terms of the existing literature, this paper is closest to Riascos and Vegh (2004). Like them, I study optimal fiscal policy in a small open economy. However, this paper differs in three key respects from their paper. First, the goal of the present paper is to characterize the behavior of optimal tax rates and government debt under incomplete markets in a small open economy, while the goal of Riascos and Vegh (2004) is to analyze the procyclicality of fiscal policy in developing countries, so they do not analyze the optimal behavior of public debt under incomplete markets. Second, these authors consider an endowment economy, while I consider a production economy with an elastic labor supply, so movements in the tax rate affect the labor supply and output. Third, they develop a model in which the interest rate is constant, while I build a model in which the interest rate is stochastic to study how interest rate shocks affect the properties of the optimal fiscal policy.

Optimal fiscal policy models for small open economies have not incorporated interest rate uncertainty, distortionary taxation and market incompleteness into a single framework. Previous papers either assume that agents have access to complete international asset markets or that interest rates are constant. In this paper, I solve the Ramsey problem for a small open economy under interest rate uncertainty, distortionary taxation and market incompleteness. Finally, by combining these elements in a general equilibrium model, I am able to characterize the properties of optimal fiscal policy in this environment, on the basis of Ramsey's principle for optimal taxation.

[5] Robert Barro (1979) hypothesized that government debt and tax rates should follow random walks, regardless of the serial correlations of the shocks that hit the economy.

3. The Model

Consider a small open economy with three agents, households, firms and a government, where the only asset traded in international financial markets is a one period real discount bond. Markets are incomplete because agents can only lend and borrow issuing and buying non contingent one period real discount bonds.

Households derive utility from consumption and leisure. Firms produce goods using labor as the only input. In addition, firms have to pay for part of the wage bill before production takes place creating a need for working capital as in Neumeyer and Perri (2005). The government finances an exogenous stochastic sequence of unproductive consumption by issuing non contingent debt and by levying income taxes at the rate τ_t.

In each period, the economy experiences one of many events s_t. We denote by $s^t = (s_0,....,s_t)$ the history of events up to and including period t. The probability, as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. The model economy is subject to three shocks: productivity shocks $z(s^t)$, interest rate shocks $R(s^t)$ and government spending shocks $g(s^t)$. It is assumed that the shocks are independent in order to isolate the effects of each shock, and that each shock follows an AR(1) process.

(1) $\log z(s^{t+1}) = \rho_z \log z(s^t) + \varepsilon_z(s^{t+1})$

(2) $\log R(s^{t+1}) = \rho_R \log R(s^t) + \varepsilon_R(s^{t+1})$

(3) $\log g(s^{t+1}) = \rho_g \log g(s^t) + \varepsilon_g(s^{t+1})$

where $q(s^t) = 1/R(s^t)$ denotes the price of a bond that pays one unit of consumption next period in every state, and $R(s^t)$ denotes the gross interest rate.
It is also assumed that the innovations are normally distributed and serially uncorrelated.

3.1 Households

There is a representative household with preferences given by the following utility function:

(4) $\sum_{t=0}^{\infty}\sum_{s^t}\beta^t u(c(s^t), h(s^t))\mu(s^t)$

where $\beta \in (0,1)$ denotes the subjective discount factor. Households derive utility from consumption and leisure. Let $c(s^t)$ denote consumption, and $h(s^t)$ hours worked. The single period utility function is assumed to be increasing in consumption, decreasing in hours, strictly concave, and twice continuously differentiable.

Each period, the representative household supplies labor in a competitive labor market, receives wage income net of taxes, profit income net of taxes from the ownership of firms, and issues one-period non-contingent real discount bonds. The household spends its income on consumption, debt repayment, and on the cost of adjusting its debt portfolio[6].

The household's budget constraint is then given by

(5) $$c(s^t) + d(s^{t-1}) + \psi(d(s^t)) \leq q(s^t)d(s^t) + (1-\tau(s^t))w(s^t)h(s^t) + (1-\tau(s^t))\Pi(s^t)$$

for all t, s^t.

where $\tau(s^t)$ denotes the tax rate, $w(s^t)$ denotes the wage rate, $\Pi(s^t)$denotes profits, $d(s^t)$ denotes debt issued by the household, and $\psi(d(s^t))$ denotes adjustment costs on debt, where $\psi(.)$ is a convex function.

In addition to this budget constraint, the household is subject to the following no-Ponzi-game condition:

(6) $$\lim_{j\to\infty} \prod_{\tau=0}^{j} q(s^{t+\tau})d(s^{t+j}) \leq 0 \quad \text{for all t, } s^t.$$

The assumptions on the utility function imply that the household will always choose state-contingent sequences such that constraints (5) and (6) hold with equality.

The household's problem is then to choose state-contingent sequences of consumption, hours and debt that maximize the expected lifetime utility (4) subject to the sequence of budget constraints (5), and the no-Ponzi-game constraints (6), for a given initial value of debt, and for the given sequences of wage rates, taxes, bond prices and profits.

The first order conditions associated with the household's maximization problem are (5) and (6) holding with equality for all t, s^t, and:

[6] Adjustment costs on debt are introduced to guarantee that the equilibrium solution of the model is stationary. See Schmitt-Grohe and Uribe (2003a) for the different methods to induce stationarity including this one.

(7) $-\frac{u_h(s^t)}{u_c(s^t)} = (1-\tau(s^t))w(s^t)$ for all t, s^t

where $u_c(s^t)$ denotes the derivative of the utility function with respect to consumption, and $u_h(s^t)$ denotes the derivative of the utility function with respect to hours.

First-order condition (7) shows that the tax rate introduces a wedge between the consumption-leisure marginal rate of substitution and the wage rate. For a given wage rate, the representative household will tend to work less the higher is the tax rate.

(8) $u_c(s^t)[q(s^t) - \psi'(d(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1})\mu(s^{t+1}/s^t)$ for all t, s^t

First-order condition (8) shows that if the representative household chooses to issue an additional unit of debt, then today's consumption increases by $q(s^t)$ minus the marginal debt adjustment cost. The expression on the left of (8) represents the utility value of this increase in consumption. Next period, the household must deliver the promised unit of consumption. The expression on the right of (8) represents the utility value of this repayment in terms of today's utility. At the optimum, the marginal benefit of an additional unit of debt must equal its marginal cost.

3.2 Firms

Firms are identical, perfectly competitive in input and goods markets and produce the final good using labor as the only input. At the beginning of the period firms hire labor to produce a final good $y(s^t)$ that will become available at the end of the period.

Firms have to pay a fraction of the wage bill at the beginning of the period. However, they have no resources to do it because production becomes available only at the end of the period. Therefore, firms have to borrow in the financial markets at the rate $R(s^t)$.

The timing of the representative firm is as follows: first it observes the realization of the state, then it decides how much to produce and requests the working capital loan to hire labor. At the end of the period, when output becomes available, firms repay the working capital loan plus interest.

Firms produce the final good using a concave technology.

(9) $y(s^t) = z(s^t) f(h(s^t))$

where $y(s^t)$ denotes output, and $z(s^t)$ is an exogenous and stochastic productivity shock.

Given the prices $w(s^t), R(s^t)$, the firm's problem is to choose labor $h(s^t)$, in order to maximize profits

$$\max_{h(s^t)} \left\{ z(s^t) f\left(h(s^t)\right) - \left[1 + \theta\left(R(s^t) - 1\right)\right] w(s^t) h(s^t) \right\}$$

where $f(.)$ is strictly increasing, strictly concave and homogeneous of degree $\eta \leq 1$, θ is the fraction of the wage bill that the firm has to pay in advance, and $R(s^t) = \frac{1}{q(s^t)}$ is the gross interest rate.

The first order condition associated with the firm's maximization problem for all t, s^t is:

$$(10) \quad w(s^t) = \frac{z(s^t) f'\left(h(s^t)\right)}{1 + \theta\left(R(s^t) - 1\right)}$$

First-order condition (10) shows that the need for working capital to finance the wage bill makes the demand for labor sensitive to the interest rate. Since firms have to borrow to pay for labor, an increase in the interest rate makes the effective labor cost higher, so the firm's labor demand falls for any given real wage.

3.3 The Government

The government faces a sequence of public consumption denoted by $g(s^t)$, which is exogenous, stochastic and unproductive. The government has access to international financial markets where it can lend and borrow only in the form of one-period real non-contingent discount bonds. Government spending is financed by taxing income, and by issuing one-period risk-free debt denoted by $b(s^t)$.

Each period, the government collects taxes on wage and profit income, and issues new debt. The government uses its income to pay for government expenditures, last period debt, and adjustment costs on its debt portfolio. The government's budget constraint is then given by

$$(11)\quad g(s^t)+b(s^{t-1})+\psi(b(s^t))\leq q(s^t)b(s^t)+\tau(s^t)w(s^t)h(s^t)+\tau(s^t)\Pi(s^t)$$

for all t, s^t

where $\psi(b(s^t))$ denotes adjustment costs on government's debt, and $\psi(.)$ is a convex function.

The government is also subject to a no-Ponzi-game condition of the form:

$$(12)\quad \lim_{j\to\infty}\prod_{\tau=0}^{j} q(s^{t+\tau})b(s^{t+j})\leq 0 \qquad \text{for all t, } s^t.$$

This constraint is a requirement for the existence of a well defined Ramsey equilibrium. The no-Ponzi game constraint cannot be ignored because without it the first best allocation is feasible. A benevolent government seeking to maximize the welfare of private agents will always choose state-contingent allocations such that (12) holds with equality. The fiscal policy consists in the announcement of a state-contingent sequence for the tax rate $\{\tau(s^t)\}_{t=0}^{\infty}$.

3.4 Competitive Equilibrium

Given initial conditions d_{-1}, b_{-1}, and exogenous stochastic processes $\{R(s^t), g(s^t), z(s^t)\}_{t=0}^{\infty}$, an equilibrium is a state-contingent sequence of allocations $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty}$, a state-contingent sequence of prices $\{w(s^t)\}_{t=0}^{\infty}$, and a state-contingent sequence of policies $\{\tau(s^t)\}_{t=0}^{\infty}$ such that:

(i) The allocations solve the firm's and the household's problems at the equilibrium prices.

(ii) Markets for factor inputs and goods clear.

(iii) The government satisfies its budget constraint.

Since this is a small open economy, the household's debt position $d(s^{t-1})$, plus the government's debt position $b(s^{t-1})$, and the working capital debt $\theta w(s^t)h(s^t)=\frac{\theta y(s^t)\eta}{1+\theta(R(s^t)-1)}$ represent the country's foreign debt position in period t.

Let $TB(s^t)$ denote the trade balance, where

(13) $TB(s^t)=y(s^t)-c(s^t)-g(s^t)-\psi(d(s^t))-\psi(b(s^t))$

The evolution of the country's foreign debt position is then given by

(14) $q(s^t)(d(s^t)+b(s^t))=d(s^{t-1})+b(s^{t-1})+\frac{\theta(R(s^t)-1)}{1+\theta(R(s^t)-1)}y(s^t)\eta-TB(s^t)$

where $\frac{\theta(R(s^t)-1)}{1+\theta(R(s^t)-1)}y(s^t)\eta=\theta(R(s^t)-1)w(s^t)h(s^t)$ represent the interests paid by firms on their working capital.

Finally, profits are given by

(15) $\Pi(s^t)=z(s^t)f(h(s^t))(1-\eta)$

4 Ramsey Problem

I follow the Ramsey approach in characterizing the optimal fiscal policy. In this approach, the Ramsey planner chooses an allocation that maximizes the household's utility subject to the condition that this allocation be implementable as a competitive equilibrium. Moreover, I assume that the Ramsey planner commits to the announced policies. To simplify the analysis of optimal fiscal policy, I adopt the approach of characterizing the equilibrium in primal form. This involves recasting all the prices and policy instruments in terms of allocations.

4.1 Primal Form of the Competitive equilibrium

The following proposition presents the primal form of the competitive equilibrium.

Proposition 1: Given initial conditions d_{-1}, b_{-1}, and exogenous stochastic processes for $\{R(s^t), g(s^t), z(s^t)\}_{t=0}^{\infty}$, state-contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty}$ satisfy (6), (8), (12) and:

$$(16)\quad c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t)d(s^t) - \frac{u_h(s^t)}{u_c(s^t)} h(s^t)\left(1 + \left[1 + \theta(R(s^t) - 1)\right]\frac{(1-\eta)}{\eta}\right)$$

$$(17)\quad \begin{aligned} &q(s^t)(d(s^t) + b(s^t)) + z(s^t) f(h(s^t))\left[\frac{\eta}{1 + \theta(R(s^t) - 1)} + (1-\eta)\right] = \\ &d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t)) \end{aligned}$$

if and only if they satisfy (5), (7), (10), (11), (15), (6), (8), (12).

Proof. See Appendix.

The Ramsey problem then consists in maximizing (4) subject to the sequence of constraints (6), (8), (12), (16) and (17), taking as given d_{-1}, b_{-1}.

Formally, the problem can be expressed as follows:

$$\max_{\substack{c(s^t), h(s^t), \\ d(s^t), b(s^t)}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c(s^t), h(s^t))\mu(s^t)$$

subject to:

$$(8)\quad u_c(s^t)\left[q(s^t) - \psi'(d(s^t))\right] = \beta \sum_{s^{t+1}} u_c(s^{t+1})\mu(s^{t+1}/s^t)$$

(16) $c(s^t)+d(s^{t-1})+\psi(d(s^t))=q(s^t)d(s^t)-\frac{u_h(s^t)}{u_c(s^t)}h(s^t)\left(1+\left[1+\theta(R(s^t)-1)\right]\frac{(1-\eta)}{\eta}\right)$

(17) $$q(s^t)(d(s^t)+b(s^t))+z(s^t)f(h(s^t))\left[\frac{\eta}{1+\theta(R(s^t)-1)}+(1-\eta)\right]=d(s^{t-1})+b(s^{t-1})+ c(s^t)+g(s^t)+\psi(d(s^t))+\psi(b(s^t))$$

(6), (12)

for all t, s^t

d_{-1}, b_{-1} given

The constraints specified above can be classified in three groups. (6) and (12) are the no-Ponzi game constraints, (16), (17) are predetermined equations because they include variables known at time t. Finally (8) is an expectational equation, because it contains a conditional expectation of future variables based on information at t, s^t.

Since implementability constraint (8) of the Ramsey problem involves a conditional expectation of future control variables, the usual Bellman equation is not satisfied, so the optimal choice at time t is not a time invariant function of the state variables $\{d_{t-1}, b_{t-1}, z_t, g_t, R_t\}$ as in standard dynamic programming, and the whole history of shocks can matter for today's optimal decision.

Nevertheless, Marcet and Marimon (1998) show that when the original maximization problem is not recursive because implementability constraints depend on plans for future variables, an equivalent recursive saddle point problem can be constructed leading to a recursive formulation. The resulting saddle point problem expands the state space by including new state variables that summarize the evolution of the lagrange multipliers of the original problem.

To solve the Ramsey problem, we need to write the problem in a recursive framework. To this end, I follow Marcet and Marimon's approach. The first step in this approach is to transform the original problem into a recursive saddle point problem.

The corresponding Lagrangian is

$$\Gamma = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\left\{u(c(s^t),h(s^t)) + \gamma(s^t)\left\{u_c(s^t)[q_t(s^t) - \psi'(d_{t+1}(s^t))] - \beta\sum_{s^{t+1}}u_c(s^{t+1})\mu(s^{t+1}/s^t)\right\}\right\}$$

subject to (6), (12), (16), (17), d_{-1}, b_{-1} given, where $\beta^t\mu(s^t)\gamma(s^t)$ is the Lagrange multiplier of (8) at t, s^t. This is still not a recursive problem, since future variables are present in the return function Γ. However, using the law of iterated expectations and reordering terms, one can show that the function H defined as

$$H = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\{u(c(s^t),h(s^t)) + \gamma(s^t)u_c(s^t)[q_t(s^t) - \psi'(d_{t+1}(s^t))] - \varsigma(s^t)u_c(s^t)\}$$

$\varsigma(s^{t+1}) = \gamma(s^t)$ for all $t \geq 0$, and $\varsigma_0 = 0$

is such that, for all feasible sequences Γ =H.

Therefore, any solution to the original Ramsey problem must also be a solution to the following saddle point problem

$$H = \min_{\gamma(s^t)}\max_{c(s^t),h(s^t)}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\{u(c(s^t),h(s^t)) + \gamma(s^t)u_c(s^t)[q_t(s^t) - \psi'(d_{t+1}(s^t))] - \varsigma(s^t)u_c(s^t)\}$$

subject to:

$\varsigma(s^{t+1}) = \gamma(s^t)$ for all $t \geq 0$, and $\varsigma_0 = 0$

$$c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t)d(s^t) - \frac{u_h(s^t)}{u_c(s^t)}h(s^t)\left(1 + [1 + \theta(R(s^t) - 1)]\frac{(1-\eta)}{\eta}\right)$$

$$q(s^t)(d(s^t)+b(s^t))+z(s^t)f(h(s^t))\left[\frac{\eta}{1+\theta(R(s^t)-1)}+(1-\eta)\right]=d(s^{t-1})+b(s^{t-1})+$$
$$c(s^t)+g(s^t)+\psi(d(s^t))+\psi(b(s^t))$$

(6) and (12)

for all t, s^t

taking as given d_{-1}, b_{-1}

Here $\varsigma(s^t)$ acts as a co-state variable. Notice that this saddle point problem does not have any future variables in the constraints and that all the functions in the constraints are known. If we include $\varsigma(s^t)$ in the set of state variables, the problem becomes recursive.

This saddle point problem has a recursive formulation, in the sense that there exists a unique value function $W(d, b, \varsigma, s)$ satisfying

$$W(d,b,\varsigma,s)=\min_{\gamma}\max_{c,h,d',b'}\{u(c,h)+\gamma u_c(c,h)[q-\psi'(d')]-\varsigma u_c(c,h)+\beta E[W(d',b',\varsigma',s')/s]\}$$

subject to

$$c+d+\psi(d')=qd'-\frac{u_h(c,h)}{u_c(c,h)}h\left(1+[1+\theta(R-1)]\frac{(1-\eta)}{\eta}\right)$$

$$q(d'+b')+zf(h)\left[\frac{\eta}{1+\theta(R-1)}+(1-\eta)\right]=d+b+c+g+\psi(d')+\psi(b')$$

$$\varsigma'=\gamma$$

where $s=(z,R,g)$ and $q=\dfrac{1}{R}$

Let χ be the policy function of this saddle point functional equation, where

$$\chi(d,b,\varsigma,s) = \arg\min_{\gamma}\max_{c,h,d',b'}\{u(c,h) + \gamma u_c(c,h)[q - \psi'(d')] - \varsigma u_c(c,h) + \beta E[W(d',b',\varsigma',s')/s]\}$$

subject to

$$c + d + \psi(d') = qd' - \frac{u_h(c,h)}{u_c(c,h)} h\left(1 + [1 + \theta(R-1)]\frac{(1-\eta)}{\eta}\right)$$

$$q(d'+b') + zf(h)\left[\frac{\eta}{1+\theta(R-1)} + (1-\eta)\right] = d + b + c + g + \psi(d') + \psi(b')$$

$$\varsigma' = \gamma$$

The solution to this functional equation yields a stationary policy function χ, so that the optimal solution to the Ramsey problem satisfies $(c_t, h_t, d_t, b_t, \gamma_t) = \chi(d_{t-1}, b_{t-1}, \varsigma_t, s_t)$ for all t and $\varsigma_0 = 0$.

This uniquely defines stationary policies for the tax rate τ_t and the wage rate w_t from (7) and (10). The solution is recursive since only the values $(d_{t-1}, b_{t-1}, \varsigma_t, s_t)$ are relevant from past history and the policy function χ is time invariant.
Dependence of the optimal solution on ς is the reason that the model is not recursive in the standard sense of having a time-invariant policy function of $\{d_{t-1}, b_{t-1}, s_t\}$.

The Lagrangian associated with the Ramsey problem, after substituting $\varsigma_t = \gamma_{t-1}$ into the objective function is given by:

$$L = \min_{\substack{\gamma_t, \lambda_t \\ \phi_t}} \max_{\substack{c_t, h_t \\ d_t, b_t}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} u(c_t, h_t) + \gamma_t u_c(t)[q_t - \psi'(d_t)] - \gamma_{t-1} u_c(t) + \lambda_t \\ \left[q_t d_t - \dfrac{u_h(t)}{u_c(t)} h_t \left(1 + \dfrac{1-\eta}{\eta} [1 + \theta(R_t - 1)] \right) - d_{t-1} - c_t - \psi(d_t) \right] \\ + \phi_t \\ \left[\begin{array}{l} q_t [d_t + b_t] + z_t f(h_t) \left[\dfrac{\eta}{1 + \theta(R_t - 1)} + (1-\eta) \right] - c_t - g_t \\ - d_{t-1} - b_{t-1} - \psi(d_t) - \psi(b_t) \end{array} \right] \end{array} \right\}$$

where $\beta^t \lambda_t$, and $\beta^t \phi_t$ are the Lagrange multipliers associated with constraints (16) and (17) respectively

The first-order conditions that characterize the optimal policy are shown in the Appendix. Since the optimality conditions of the Ramsey problem cannot be solved analytically, I compute the equilibrium dynamics by solving a log-linear approximation to the Ramsey planner's optimality conditions.

5 Dynamic Properties of the Optimal Policy.

In this section, I carry out some simulations to study the dynamic properties of the model economy under the Ramsey policy with incomplete markets. I compute the equilibrium dynamics by solving a log-linear approximation to the Ramsey planner's optimality conditions. First, I present the baseline calibration of the model. Second, I show and discuss the impulse responses of the model to the different shocks that hit the economy to illustrate the dynamic properties of the optimal fiscal policy. Third, I present and analyze the second moments of the simulated time series. Finally, I conduct a sensitivity analysis to evaluate how the results change when we vary some of the parameter values.

5.1 Calibration

The benchmark model is calibrated so as to make it consistent with some of the empirical regularities that reflect the structure of a typical emerging economy. In particular, data from Mexico is used to calibrate the parameters of the benchmark model. The data considered corresponds to quarterly observations for the period 1980-2004. In general the results are robust to changes in the parameters, therefore, I only report sensitivity analyses for those parameters that are crucial for determining the effects of the shocks in the model economy.

The time endowment, which can be divided between labor and leisure is normalized to 1. The utility function follows the GHH specification[7].

$$u\left(c\left(s^t\right), h\left(s^t\right)\right) = \frac{\left(c\left(s^t\right) - \varphi h\left(s^t\right)^{\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

The parameter σ, the coefficient of relative risk aversion, is set equal to 2, a value that is standard in the literature. The parameter ν is set to 1.6 following Neumeyer and Perri (2005). This parameter determines the labor supply elasticity, which equals $\frac{1}{\nu - 1}$. I set φ to match an average time spent working of 20% of total time, which is a standard value. I set β to match an average real interest rate in Mexico of 10.17% per year.[8]

I specify the following production function

$$y\left(s^t\right) = z\left(s^t\right) f\left(h\left(s^t\right)\right)$$

where

$$f\left(h\left(s^t\right)\right) = h\left(s^t\right)^{\eta}$$

I choose a value for η consistent with a labor share of income of 0.67, a standard value. The labor's share of income in the model is equal to $\frac{\eta}{1+\theta\left(\overline{R}-1\right)}$ and $\overline{R}$ is the average gross interest rate. It is also assumed that all the wage bill is paid in advance, and therefore, I set $\theta = 1$.

[7] This specification has the property that the marginal rate of substitution between consumption and leisure is independent of consumption. Therefore, labor supply is not affected by consumption.

[8] Average real interest rate in Mexico from 1994 to 2002. See Neumeyer and Perri (2005).

I also assume that the adjustment cost functions for the household and the government are respectively.

$$\psi\left(d\left(s^t\right)\right)=\frac{\psi}{2}\left(d\left(s^t\right)-\bar{d}\right)^2$$

$$\psi\left(b\left(s^t\right)\right)=\frac{\psi}{2}\left(b\left(s^t\right)-\bar{b}\right)^2$$

where ψ is a constant determining the size of the debt holding costs and $\bar{d},\bar{b}$ are the steady state values of the household debt and government debt respectively. The parameter ψ is chosen so that the costs are minimal and do not affect the short-run properties of the model. Therefore, ψ is set to the minimum value that guarantees that the equilibrium solution is stationary.

The steady state debt of the government in the model is not uniquely determined by the parameters of the model, so I set $\bar{b}$ to match the average public-debt to GDP ratio in Mexico for 1980-2004, which in the data is 37.5%.[9] I set $\bar{d}$ to match the average of the ratio between the net debt foreign position and output in Mexico, which in the data is approximately equal to 40%, and in the model corresponds to:[10]

$$\frac{\bar{d}}{\bar{y}}+\frac{\bar{b}}{\bar{y}}+\frac{\eta\theta}{1+\theta\left(\bar{R}-1\right)}$$

where $\frac{\eta\theta}{1+\theta\left(\bar{R}-1\right)}=\frac{\theta\bar{w}\bar{h}}{\bar{y}}$

I choose a value for $\bar{g}$ consistent with the average share of government spending in output in Mexico for 1980-2004, which in the data is approximately equal to 13.5% .[11]

[9] Source "Deuda Económica Amplia del Sector Público: Banco de México"
[10] The average net foreign position of Mexico is computed by averaging foreign asset positions data, constructed using cumulated capital flows, from 1983 to 1998, as reported in Lane and Milesi-Ferreti (2001).
[11] Source: INEGI

It is assumed that the exogenous processes for productivity, interest rates and government expenditures follow independent AR(1) processes. First, I remove a linear trend from the variables. Then, I estimate the parameters of the stochastic processes by OLS.
The parameters of the benchmark model are shown in the following table.

Table 1: Parameter Values

Parameter	*Value*
β	0.9761
σ	2
ν	1.6
φ	1.4367
η	0.68
ψ	0.0008
ρ^z	0.89
ρ^g	0.87
ρ^R	0.61
σ_ε^z	0.021
σ_ε^g	0.07
σ_ε^R	0.017

5.2 Impulse Responses

In this section I present and discuss the impulse responses of the model in order to illustrate the dynamic properties of the optimal fiscal policy. The interest rate is expressed in percentage points. Output, productivity, hours worked, the wage rate, the tax rate, tax revenues and government expenditures are expressed in percentage deviations from their steady state values. The deviations of government debt, the primary deficit, the total deficit,. the trade balance, the current account and consumption are expressed as a percent of steady state output.

Interest Rate Shocks

Figure 1 displays the optimal responses of the variables in the model to a 100 basis points increase in the interest rate. In response to a positive and persistent increase in the interest rate, labor demand shifts to the left on impact and its effect on hours worked depends on the slope of the labor supply an on its reaction to an interest rate innovation. Since I assumed GHH preferences, the labor supply is independent of the level of consumption, however, it is not independent of the interest rate because labor supply depends on the tax rate, and the tax rate is affected by interest rate shocks. In response to a positive innovation in the interest rate, the tax rate decreases on impact, and consequently, the labor supply shifts to the right on impact. Therefore, a positive innovation in the interest rate generates a reduction in the wage rate.

Since the optimal fiscal policy smoothes tax distortions over time, the response of the tax rate is very small, which implies that the labor demand shifts more than the labor supply, and as result hours worked and output also decrease on impact. The magnitude of the fall in hours depends on the wage elasticity of labor demand and on the wage elasticity of labor supply which are respectively $\frac{1}{\eta-1}$ and $\frac{1}{\nu-1}$. The lower is ν, or the higher is η, the more labor decreases in response to an interest rate shock.

Interest rate shocks have two effects on consumption. The direct effect reduces consumption, because an increase in the interest rate makes future consumption cheaper. The indirect effect is through labor. Since GHH preferences are not separable across consumption and hours, a fall in hours worked makes the household reduce consumption as well in order to keep the marginal utility of consumption smooth over time. The sum of the two effects makes the response of consumption to an interest rate shock to be larger than the response of output. Since consumption falls more than output on impact, and government expenditures remain constant, the trade balance, which is given by equation (13), increases.

For a given tax rate, wage income and profit income decrease. Since taxes are distortionary, a lower household income, and therefore, a lower tax base, induces the government to increase its debt, and to decrease the tax rate in order to smooth tax distortions over time. Tax revenues also decrease, not only because the tax rate decreases, but also because the tax base which includes wage income and profit income decreases. The primary deficit increases as well since tax revenues decrease and government expenditures remain constant. The deficit which includes the primary deficit plus interest payments also increases, not only because the primary deficit increases but also because interest payments on debt increase as a consequence of the interest rate increase.

Public debt responds on impact less to the shock than the other variables, but it accumulates over time. The responses of debt and the primary deficit to an interest rate shock have the same sign in the first periods. However, the response of the primary deficit changes sign after a few periods, because a higher debt interest will have to be serviced in the future since debt accumulates over time. The impulse response functions of the tax rate and tax revenues also change sign after a few periods in order to prevent debt from exploding in the long run. Moreover, the impulse response functions of public debt and the tax rate are more persistent than the impulse response functions of the other variables in the economy. Finally, the current account decreases since the economy's net foreign asset position deteriorates.

In the long-run (not shown in the diagrams) all variables converge to the steady state.

The optimal response of the tax rate to an aggregate shock is given by first order condition (7). If we log-linearize this first order condition, we have that the optimal percentage deviation of the tax rate is given by

$$\tilde{\tau}_t = \frac{1-\bar{\tau}}{\bar{\tau}} * \left[\tilde{z}_t - \frac{\theta \bar{R}}{1+\theta\left(\bar{R}-1\right)} \tilde{R}_t - (\nu - \eta)\tilde{h}_t \right]$$

where $\bar{\tau}$ is the steady state value of the tax rate.

Since I assumed that $\theta = 1$, and because we are considering interest rate shocks, the above equation simplifies to:

$$\tilde{\tau}_t = \frac{1-\bar{\tau}}{\bar{\tau}} * \left[-\tilde{R}_t - (\nu - \eta)\tilde{h}_t \right]$$

Interest rate shocks have two effects on the tax rate. One direct effect, and one indirect effect through the response of labor to interest rate shocks. The direct effect decreases the tax rate, however, since labor decreases, the indirect effect of the interest rate through labor will tend to increase the tax rate. Since the two effects go in opposite directions, the response of the tax rate will tend to be very small. With the benchmark parameterization, the direct effect dominates, so the tax rate decreases after a positive interest rate shock.

Productivity Shocks

Figure 2 displays the impulse response functions of the variables in the model to a 1 percent decrease in productivity. Since taxes are distortionary, the optimal response of the government is to increase its debt, and to have a small but persistent increase in the tax rate that will pay off the increase in the stock of debt gradually over time. Tax revenues also decrease, even though the tax rate has increased, since the tax base decreases significantly. The primary deficit increases as well since tax revenues decrease and government expenditures remain constant.

The immediate response of public debt to the shock is again small compared to the other variables but it accumulates over time. The responses of debt and the primary deficit to a fall in productivity have the same sign in the first periods. However, the response of the primary deficit changes sign in the future in order to pay for the additional debt service.

In response to the fall in productivity, labor demand shifts to the left on impact. Since the tax rate increases, and labor supply is independent of consumption, labor supply also shifts to the left. Consequently, hours worked and output fall on impact. Labor demand shifts more than labor supply because the increase in the tax rate is very small, therefore, the wage rate also decreases. Since consumption responds less than output and government expenditures remains constant, the trade balance deteriorates. The current account decreases because the economy's net foreign asset position deteriorates.

The impulse response functions of public debt and the tax rate are again more persistent than the impulse response functions of the other variables in the economy. In the long-run (not shown in the diagrams) all variables converge to the steady state.

The optimal response of the tax rate to a fall in productivity is now given by

$$\tilde{\tau}_t = \frac{1-\bar{\tau}}{\bar{\tau}} * \left[\tilde{z}_t - (\nu - \eta)\tilde{h}_t\right]$$

Productivity shocks have two effects on the tax rate. One direct effect, and one indirect effect through the response of labor to productivity shocks. The direct effect decreases the tax rate after a fall in productivity, however, since labor decreases, the indirect effect will tend to increase the tax rate. Since the two effects go in opposite directions, the response of the tax rate will tend to be very small. With the benchmark parameterization, the indirect effect dominates, so the tax rate increases after a negative productivity shock

Government Spending Shocks

Figure 3 displays the impulse response functions to a 1 percent increase in government spending. Given tax rates, household's demands for consumption and leisure are unaffected by government spending shocks, therefore the government should finance the increase in its expenditure by the least distortionary combination of tax rates and government debt.

In response to a positive and persistent increase in government spending the government finds it optimal to increase its debt, and to have a small but persistent increase in the tax rate that will pay off the increase in the stock of debt gradually over time. Tax revenues increase, but by less than the increase in government expenditure, therefore, the primary deficit increases. Public debt responds again on impact less to the shock than the other variables, but it accumulates over time. The responses of debt and the primary deficit to an increase in

government spending have the same sign in the first periods. However, the response of the primary deficit changes sign in the future in order to pay for the additional debt service.
Due to the fact that the tax rate increases, labor supply shifts to the left on impact, and since labor demand is independent of the tax rate, the wage rate increases and hours worked decrease. Consequently output and consumption also decrease. The trade balance deteriorates, but it changes sign after some periods because a higher debt interest will have to be serviced in the future in response to the increase in public debt. Finally, the current account decreases since the economy's net foreign asset position deteriorates. In the long run (not shown in the diagrams) all variables converge to the steady state.

5.3 Second Moments and Simulations

The following table displays a number of sample moments of key macroeconomic variables under the Ramsey policy. The moments are computed as follows. First, I generate simulated time series of length 200 for the variables of interest, second, I HP-filter the simulated time series, and finally, I compute the moments. I repeat this procedure 500 times and then compute the average of the moments. Table 2 reports the volatility, correlations and autocorrelations calculated with the procedure described above. All parameters are the ones of the benchmark calibration.

Table 2

			Prod.Shock	Gov.Shock	Int.Shock	Output
Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	24.71	0.94	-0.07	0.17	0.11	-0.16
Tax rate	2.96	0.70	-0.19	0.85	-0.19	-0.24
Output	4.83	0.63	0.95	-0.08	-0.25	1
Cons.	4.04	0.57	0.71	-0.22	-0.63	0.88
Hours	3.59	0.60	0.82	-0.18	-0.50	0.95

Some interesting facts emerge from this table:

1) The income tax rate and specially the public debt are quite persistent. The reason is that the planner finances an increase in government spending or a decrease in productivity partly by increasing public debt and partly by increasing the tax rate. In order to avoid a large distortion at the time of the shock, the planner smoothes the tax

increase over time. As a consequence, the stock of public debt displays a persistent increase.

2) The Ramsey planner keeps distortionary taxes smooth over the business cycle; the standard deviation of taxes is smaller than the standard deviation of the other variables in the economy. The volatility of public debt serves primarily the purpose of smoothing the process of income tax distortions.

3) Public debt is negatively correlated with productivity and output, and positively correlated with government spending and the interest rate. By contrast, when agents have access to complete markets as in Schmitt-Grohé and Uribe (2003b), the real value of public debt is positively correlated with productivity and output, and negatively correlated with government spending. In response to a positive government spending shock the value of real government liabilities with which the government leaves the period declines, and then it converges back to its steady-state level gradually. The Ramsey planner finances all innovations to government purchases with state-contingent payments from the rest of the world. In a real economy with only risk-free debt, however, the planner has no alternative but to smooth the cost of raising distortionary revenues. Therefore, public debt and the tax rate increase in response to a positive government spending shock.

4) The tax rate is positively correlated with government spending, while output, consumption and hours are negatively correlated with government spending. By contrast, when agents have access to complete markets neither the Ramsey real allocation nor its associated income tax rate adjust in response to government spending shocks.

5) The negative correlation between consumption and government spending illustrates the limited insurance role played by non-contingent debt. If agents in a small open economy have access to complete markets as in Schmitt-Grohé and Uribe (2003b), the real allocation and the optimal tax rate are uncorrelated with government spending shocks. Government spending shocks have only wealth effects, so agents can insure completely against these shocks via international financial markets. When markets are incomplete in a small open economy, the neutrality of government spending shocks disappears because agents cannot hedge against these shocks.

6) Output, consumption and hours worked are negatively correlated with the interest rate. Since firms have to borrow to pay a fraction of the wage bill at the beginning of the period, labor demand decreases in response to an increase in the interest rate, and as a result hours worked, output and consumption also decrease.

5.4 Sensitivity Analysis

In this section, I conduct a sensitivity analysis to evaluate how the results change when we vary some of the parameter values. First, I consider the elements of the model that are crucial for determining the effects of interest rate fluctuations on the Ramsey allocation. These elements are the wage elasticity of labor supply $\frac{1}{\nu - 1}$, and the fraction of the wage bill that firms have to pay in advance θ. Second, I conduct a sensitivity analysis on the parameter values that determine the serial correlation properties of the shocks. In particular, I evaluate how the results change when we consider i.i.d shocks instead of the serially correlated shocks of the baseline calibration. In all the cases I keep all the other parameters at their baseline value, and I only analyze the impulse response functions of the model to interest rate shocks.

5.4.1 Sensitivity Analysis: Wage elasticity of labor supply $\frac{1}{\nu - 1}$

In this section, I evaluate how the results change when we reduce ν from its baseline value of 1.6 to 1.1. Notice that the lower ν the higher is the wage elasticity of labor supply. Figure 4 displays the impulse response functions to a 1 percentage point increase in the interest rate.

As the wage elasticity of labor supply increases, hours worked and output decrease more in response to an interest rate shock, therefore, for a given tax rate the tax base decreases more than in the benchmark model.

Since the government smoothes distortions over time, public debt and the deficit increase. Public debt accumulates more over time than in the benchmark model because a higher deficit needs to be financed, since the tax base decreases more in this experiment. The tax rate decreases less on impact to an interest rate shock, because public debt will be higher in the future than in the benchmark model, and consequently, higher debt interests will have to be paid. Therefore, the government optimally chooses not to decrease the tax rate as much as in the benchmark model in order to smooth tax distortions over time, and to avoid the need to increase the tax rate dramatically in the future.

The current account also decreases more than in the benchmark model because the net foreign asset position deteriorates more in this experiment. The immediate response of debt is again small compared to the other variables, but it accumulates more in this experiment than in the benchmark model because the tax base decreases by more, and therefore, a higher deficit needs to be financed. The response of the tax rate changes sign after a few periods, and it also increases more in the future than in the benchmark model to prevent debt from exploding since debt grows more in this example.

The following table reports the volatility, correlations and autocorrelations of key macroeconomic variables under the Ramsey policy for a value of ν of 1.1.

Table 3

Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	30.39	0.93	-0.11	0.10	0.12	-0.24
Tax rate	3.75	0.69	-0.55	0.72	-0.06	-0.63
Output	8.27	0.62	0.92	-0.16	-0.33	1
Cons.	7.55	0.60	0.81	-0.24	-0.49	0.97
Hours	8.76	0.61	0.84	-0.21	-0.46	0.98

The simulations shown in table 3 reveal some interesting results.

1) Increasing the wage elasticity of labor supply, increases the volatilities of all the variables. We can see from Table 2 that the standard deviations increase when we decrease ν. The reason is that an increase in the interest rate paid by firms on working capital induces a fall in hours that depends mainly on the wage elasticity of the labor demand $\frac{1}{\eta - 1}$ and on the wage elasticity of the labor supply $\frac{1}{\nu - 1}$. The lower is ν or the higher is η, the more hours worked fluctuate in response to interest rate shocks, and as a consequence the more output and consumption fluctuate as well. If the volatility of hours worked increases, then the tax base becomes more volatile, consequently, the volatilities of the tax rate and of the public debt also increase.

2) The correlations of the tax rate and of the public debt with the interest rate increase when we increase the wage elasticity of labor supply. The reason is that hours worked and output decrease more in response to positive interest rate shocks when we increase the wage elasticity of labor supply. Therefore, the tax base decreases more than in the benchmark model, and since the government smoothes tax distortions over time, public debt also increases more than in the benchmark model.

5.4.2 Sensitivity Analysis: Fraction of the wage bill paid in advance θ

In this section, I evaluate how the results change when we reduce θ from 1 to 0.5. Now, the firm has to pay only half of the wage bill in advance. Figure 5 displays the impulse response functions to a 1 percentage point increase in the interest rate.

As θ decreases from 1 to 0.5, the negative impact of interest rates on labor demand decreases, and therefore, hours worked and output decrease less than in the benchmark model in response to an interest rate shock. The primary deficit and the total deficit also increase less in response to an interest shock because tax revenues decrease less in this experiment. Public debt accumulates less than in the benchmark model because the negative effect of an interest rate shock in the economy decreases when θ decreases. The current account also decreases less than in the benchmark model because the net foreign asset position deteriorates less in this experiment. Finally, as θ decreases, the volatility of the tax rate decreases. The tax rate decreases less on impact, and it increases less in the future.

In this experiment the government can keep the tax rate smoother than in the benchmark model due to the fact that the impact of interest rate shocks in the economy decreases. The basic message of the benchmark model does not change. The government uses the optimal combination of debt and tax rates consistent with competitive equilibrium that minimizes distortions over time.

The following table 4 reports the volatility, correlations and autocorrelations of key macroeconomic variables under the Ramsey policy for a value of θ of 0.5.

Table 4

Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	24.2	0.94	-0.06	0.16	0.07	-0.13
Tax rate	2.90	0.70	-0.18	0.89	-0.08	-0.28
Output	4.64	0.65	0.98	-0.10	-0.13	1
Cons.	3.65	0.60	0.78	-0.24	-0.52	0.88
Hours	3.19	0.63	0.92	-0.21	-0.28	0.97

The simulations shown in table 4 also reveal some interesting results.

1) The standard deviations of all the variables decrease when we decrease θ. This is because by reducing θ we reduce the fraction of the wage bill that the firm has to pay on advance, consequently, the negative impact of interest rates on labor demand decreases, and therefore, hours worked and output decrease less than in the benchmark model in response to an interest rate shock. Since the volatility of the tax base decreases, the volatilities of the tax base and of the public debt decrease as well.

2) The absolute values of the correlations of hours worked, output and consumption with the interest rate decrease when we decrease θ, because the negative impact that interest rates have on labor demand decreases. Since the tax base becomes less correlated with the interest rate, the absolute value of the correlations of public debt and of the tax rate with the interest rate also decrease.

5.4.3 Sensitivity Analysis: Serial Correlation Properties of the Shocks

In this section, I evaluate how the results change when we consider serially uncorrelated shocks. Table 5 reports the volatility, correlations and autocorrelations of key macroeconomic variables under the Ramsey policy when interest rate shocks are uncorrelated, table 6 shows the case of uncorrelated productivity shocks and table 7 the case of uncorrelated government spending shocks.

Table 5: Uncorrelated Interest Rate Shocks $\rho^{R} = 0$.

Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	24.34	0.94	-0.07	0.19	0.12	-0.12
Tax rate	3.08	0.62	-0.18	0.84	-0.35	-0.20
Output	4.72	0.63	0.96	-0.1	-0.2	1
Cons.	3.42	0.53	0.84	-0.28	-0.43	0.94
Hours	3.39	0.53	0.87	-0.21	-0.43	0.96

Table 6: Uncorrelated Productivity Shocks $\rho^{z} = 0$.

Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	23.06	0.93	-0.09	0.19	0.13	-0.22
Tax rate	2.95	0.7	-0.02	0.87	-0.2	-0.1
Output	3.76	0.01	0.93	-0.15	-0.33	1
Cons.	3.18	0.36	0.48	-0.3	-0.8	0.77
Hours	2.96	0.18	0.74	-0.26	-0.61	0.93

Table 7: Uncorrelated Government Spending Shocks $\rho^{g} = 0$.

Variable	*Std dev %*	*Autocorr.*	*Corr(x,z)*	*Corr(x,g)*	*Corr(x,r)*	*Corr(x,y)*
Public debt	18.73	0.9	-0.09	0.19	0.16	-0.16
Tax rate	1.25	0.69	-0.49	0.17	-0.47	-0.38
Output	4.87	0.64	0.97	-0.01	-0.23	1
Cons.	3.93	0.56	0.75	-0.01	-0.64	0.89
Hours	3.53	0.6	0.86	-0.01	-0.49	0.96

The moments in tables 5, 6 and 7 reveal the same result as before: the optimal fiscal policy smoothes tax distortions over time. Public debt is very persistent irrespective of the degree of autocorrelation of the underlying shocks.

The results also show that the income tax rate is quite persistent independently of the assumed processes for the shocks generating aggregate fluctuations; this reflects the planner's desire to smooth the cost of raising taxes over time. The planner finances any innovation to government purchases or to the tax base partly by increasing public debt and partly by increasing the tax rate. Therefore, public debt plays an important role as a shock absorber in the absence of state-contingent debt.

6. Conclusions

I have characterized the properties of optimal fiscal policy in a small open economy where interest rates, government spending and productivity are stochastic, taxes are distortionary and markets are incomplete. This paper provides a new framework to analyze optimal fiscal policy in small open emerging economies, and extends the existing literature on optimal fiscal policy by studying the case of incomplete markets in a small open economy under uncertainty and distortionary taxation.

The main contributions of the paper are the following. First, I solve the Ramsey problem for a small open economy with incomplete markets and stochastic interest rates, government spending and productivity. I show that if we restrict the agents in a small open economy to buy and sell only one-period non-contingent real bonds, the Ramsey planner is confronted with stochastic sequences of implementability constraints that arise from the requirement that the debt be risk-free. Since conditional expectations of future variables appear in these implementability constraints, the Ramsey problem is not recursive. However, I show that is possible to recover a recursive formulation using the recursive contracts approach of Marcet and Marimon (1998).

Second, I show that the optimal fiscal policy consists in smoothing tax distortions over time. The income tax rate, and specially the public debt are very persistent irrespective of the degree of autocorrelation of the processes for the shocks generating aggregate fluctuations. This reflects the planner’s desire to smooth the cost of raising taxes over time. The Ramsey planner finances an increase in government spending or a decrease in productivity partly by increasing debt and partly by increasing the tax rate. In order to avoid a large distortion at the time of the shock, the planner smoothes the tax increase over time. As a consequence, the stock of public debt displays a persistent increase. Debt plays in this model an important role as a shock absorber. After a positive innovation in the interest rate or in government spending, or after negative innovation in productivity, the level of public debt and the primary deficit increase. Government debt responds on impact less than the other variables, but it accumulates over time, and displays the most persistent impulse response function. The responses of debt and the primary deficit have the same sign in the first periods. However, the response of the primary deficit changes sign after a few periods, because a higher debt interest will have to be serviced in the future in response to an increase in debt today. Finally, I show that debt shows more persistence than the shock processes irrespective of the degree of autocorrelation of the underlying shocks.

By contrast, if agents in a small open economy have access to complete markets as in Schmitt-Grohé and Uribe (2003b), the real allocation and the optimal tax rate are uncorrelated with government spending shocks. Government spending shocks have only wealth effects, so agents can insure completely against these shocks via international financial markets. When markets are incomplete in a small open economy, the neutrality of government spending shocks disappears because agents cannot hedge against these shocks.

In an economy with complete markets, the optimal fiscal policy completely smoothes tax distortions over time and across states of nature. Moreover, under complete markets debt and taxes are time invariant functions of the shocks affecting the economy, and hence they inherit the serial correlation properties of $\{s_t\}$, so they have the same persistence than other variables in the economy as in Lucas and Stokey (1983).

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Appendix

Proof of Proposition 1

Proposition 1: Given initial conditions d_{-1}, b_{-1}, and exogenous stochastic processes for $\{R(s^t), g(s^t), z(s^t)\}_{t=0}^{\infty}$, state-contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty}$ satisfy (6), (8), (12) and

(16) $$c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t)d(s^t) - \frac{u_h(s^t)}{u_c(s^t)} h(s^t)\left(1 + \left[1 + \theta(R(s^t) - 1)\right]\frac{(1-\eta)}{\eta}\right)$$

(17) $$\begin{aligned} &q(s^t)(d(s^t) + b(s^t)) + z(s^t) f(h(s^t))\left[\frac{\eta}{1 + \theta(R(s^t) - 1)} + (1-\eta)\right] = \\ &d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t)) \end{aligned}$$

for all t, s^t

if and only if they satisfy (5), (7), (10), (11), (15), (6), (8), (12).

First, I show that state-contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty}$ satisfying (5), (7), (10), (11), (15), (6), (8), and (12) also satisfy (16), (17), (6), (8), (12).

To obtain (16), solve (10) for $w(s^t)$, (15) for $\Pi(s^t)$ and (7) for $\tau(s^t)$. Then, use the resulting expressions to eliminate $w(s^t)$, $\Pi(s^t)$ and $\tau(s^t)$ from (5). The resulting equation is (16).

To obtain (17), substitute (5) into (11), then, solve (10) for $w(s^t)$, and (15) for $\Pi(s^t)$, and use the resulting expressions to eliminate $w(s^t)$, $\Pi(s^t)$ from this equation. The resulting expression is (17).

Now, it must be shown that if state-contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty}$ satisfy (6), (8), (12), (16) and (17), then they are also consistent with (5), (7), (10), (11), (15), (6), (8), (12).

Set $w(s^t)$ such that (10) holds, $\tau(s^t)$ such that (7) holds and $\Pi(s^t)$ such that (15) holds. Therefore, (7), (10) and (15) are satisfied by construction.

Using the definitions of $w(s^t)$, $\Pi(s^t)$ and $\tau(s^t)$ in (16), we can recover (5).

Then, use the definitions of $w(s^t)$, $\Pi(s^t)$ and $\tau(s^t)$ in (16), and the definitions of $w(s^t)$, $\Pi(s^t)$ in (17), and then combine the resulting expressions to show that (11) is also satisfied. Q.E.D.

Ramsey Problem

The first order conditions of the Ramsey problem are given by:

(18)
$$u_c(t)+\gamma_t u_{cc}(t)[q_t-\psi'(d_t)]-\gamma_{t-1}u_{cc}(t)-\lambda_t \left[\left(1+\frac{1-\eta}{\eta}[1+\theta(R_t-1)]\right)\left(\frac{u_{ch}(t)}{u_c(t)}h_t-\frac{u_h(t)u_{cc}(t)}{(u_c(t))^2}h_t\right)+1\right]-\phi_t=0$$

(19)
$$u_h(t)+\gamma_t u_{ch}(t)[q_t-\psi'(d_t)]-\gamma_{t-1}u_{ch}(t)-\lambda_t\left(1+\frac{1-\eta}{\eta}[1+\theta(R_t-1)]\right)\left(\frac{u_h(t)}{u_c(t)}+\frac{u_{hh}(t)}{u_c(t)}h_t-\frac{u_h(t)u_{ch}(t)}{(u_c(t))^2}h_t\right)+\phi z_t f'(h_t)\left(\frac{\eta}{1+\theta(R_t-1)}+(1-\eta)\right)=0$$

(20)
$$q_t d_t-\frac{u_h(t)}{u_c(t)}h_t\left(1+\frac{1-\eta}{\eta}[1+\theta(R_t-1)]\right)-d_{t-1}-c_t-\psi(d_t)=0$$

(21)
$$q_t[d_t+b_t]+z_t f(h_t)\left[\frac{\eta}{1+\theta(R_t-1)}+(1-\eta)\right]-c_t-g_t-d_{t-1}-b_{t-1}-\psi(d_t)-\psi(b_t)=0$$

(22)
$$u_c(t)[q_t-\psi'(d_t)]-\beta E_t[u_c(t+1)]=0$$

(23) $$-\gamma_t u_c(t)\psi''(d_t)+(\lambda_t+\phi_t)(q_t-\psi'(d_t))-\beta E_t[\lambda_{t+1}+\phi_{t+1}]=0$$

(24) $$\phi_t(q_t-\psi'(b_t))-\beta E_t[\phi_{t+1}]=0$$

We also have the transversality conditions

(25) $$\lim_{t\to\infty}\frac{d_t}{\prod_{j=0}^{t}R_j}=0$$

(26) $$\lim_{t\to\infty}\frac{b_t}{\prod_{j=0}^{t}R_j}=0$$

(27) $$\lim_{t\to\infty}\frac{\gamma_t}{\prod_{j=0}^{t}R_j}=0$$

d_{-1}, b_{-1} given and $\gamma_{-1}=0$

Let's denote the vector of endogenous state variables of the optimal problem by $x_{t-1}=[d_{t-1}, b_{t-1}, \gamma_{t-1}]'$. The vector of all other endogenous variables is denoted by $a_t=[c_t, h_t, \lambda_t, \phi_t]'$, and the vector of exogenous state variables by $s_t=[z_t, R_t, g_t]'$.
Then, the first order conditions that characterize the optimal fiscal policy can be represented as a system of equations of the form:

$$G(x_t, x_{t-1}, a_t, s_t)=0$$

$$E_t\left[V\left(x_{t+1}, x_t, x_{t-1}, a_{t+1}, a_t, s_{t+1}, s_t\right)\right] = 0$$

The first set is formed by the deterministic equations (18), (19), (20) and (21), and the second set is formed by the expectational equations (22), (23) and (24).

The first order conditions of the Ramsey problem cannot be solved analytically, therefore, I will compute the equilibrium dynamics by solving a log-linear approximation to the Ramsey planner's optimality conditions.

The computational approach involves two steps. First, we compute the non-stochastic steady state which is given by $G\left(\bar{x}, \bar{x}, \bar{a}, \bar{s}\right) = 0$, and $V\left(\bar{x}, \bar{x}, \bar{x}, \bar{a}, \bar{a}, \bar{s}, \bar{s}\right) = 0$.

Second, we log-linearize the above system of equations around the non-stochastic steady state and calculate the local dynamic behavior of the endogenous variables given the specified law of motion for the exogenous state variables.

The log-linearized equilibrium conditions can be written in the following form

(28) $\alpha_o \hat{x}_t + \alpha_1 \hat{x}_{t-1} + \beta_0 \hat{a}_t + \delta_0 \hat{s}_t = 0$

(29) $E_t\left[\alpha_2 \hat{x}_{t+1} + \alpha_3 \hat{x}_t + \alpha_4 \hat{x}_{t-1} + \beta_1 \hat{a}_{t+1} + \beta_2 \hat{a}_t + \delta_1 \hat{s}_{t+1} + \delta_2 \hat{s}_t\right] = 0$

(30) $\hat{s}_{t+1} = \Phi \hat{s}_t + \varepsilon_{t+1}$, $E_t\left[\varepsilon_{t+1}\right] = 0$

where the "hated" variables represent percentage deviations from the steady state, and the $\alpha_i\text{'s}, \beta_i\text{'s}$, and $\delta_i\text{'s}$ are matrices collecting the coefficients.

$$\Phi = \begin{pmatrix} \rho_z & 0 & 0 \\ 0 & \rho_R & 0 \\ 0 & 0 & \rho_g \end{pmatrix}, \text{ and } \varepsilon_{t+1} = \left[\varepsilon^z_{t+1}, \varepsilon^R_{t+1}, \varepsilon^g_{t+1}\right]'.$$

We seek processes $\{\hat{x}_t, \hat{a}_t\}$ that are consistent with (28), (29), (30) for all $\{\hat{x}_{t-1}, \hat{s}_t\}$, with the initial conditions, and with the transversality conditions. That is, we are seeking a recursive equilibrium law of motion of the form

$$\hat{x}_t = A\hat{x}_{t-1} + B\hat{s}_t$$

$$\hat{a}_t = C\hat{x}_{t-1} + D\hat{s}_t$$

$$\hat{s}_{t+1} = \Phi\hat{s}_t + \varepsilon_{t+1}$$

Therefore, we need to solve for matrices A, B, C, D, so that the equilibrium described by these rules is stable. The recursive equilibrium law of motion can be solved by the method of undetermined coefficients.

Figure 1: Impulse Responses to an Interest Rate Shock.

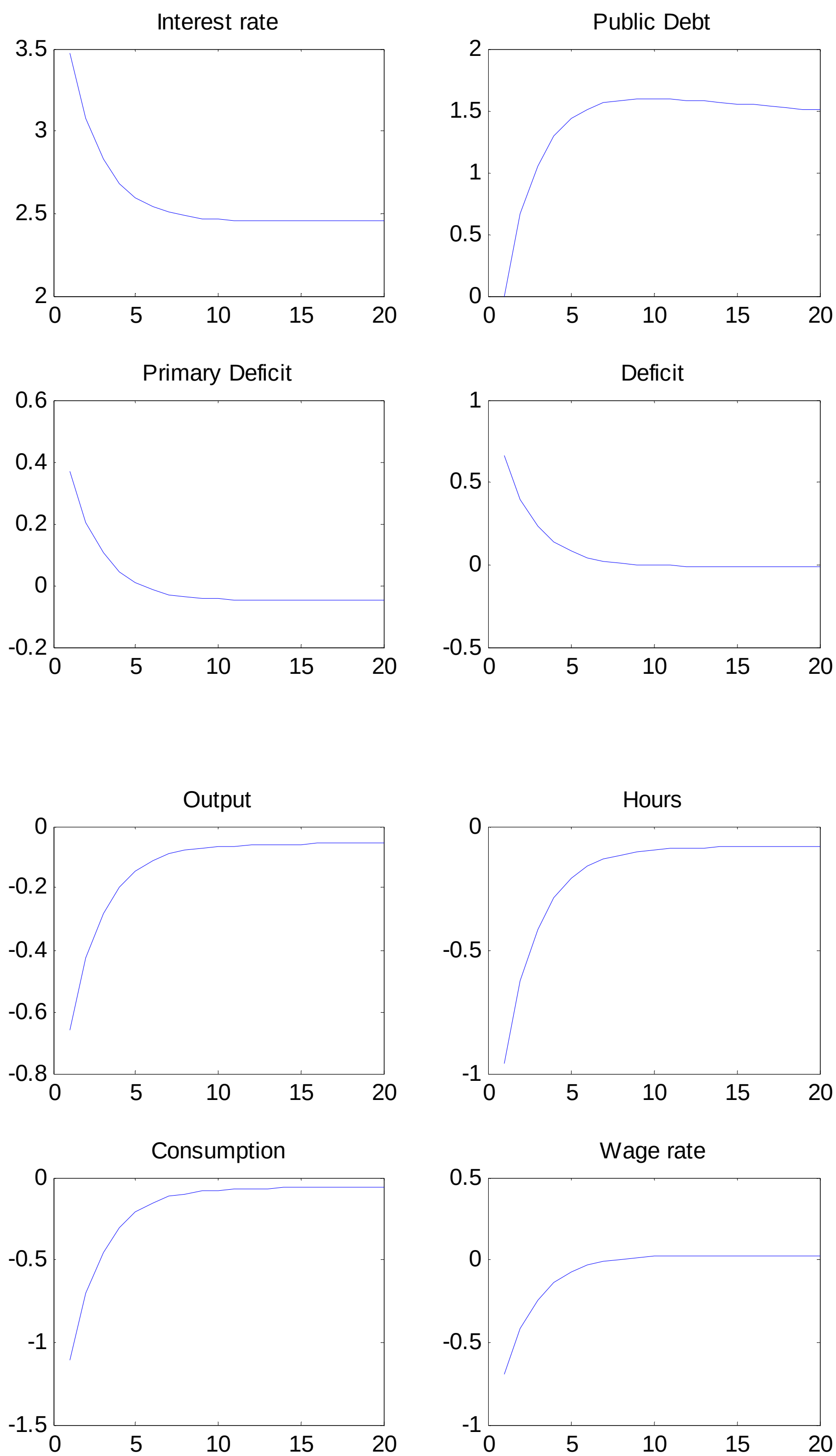

Figure1: Impulse Responses to an Interest Rate Shock (cont)

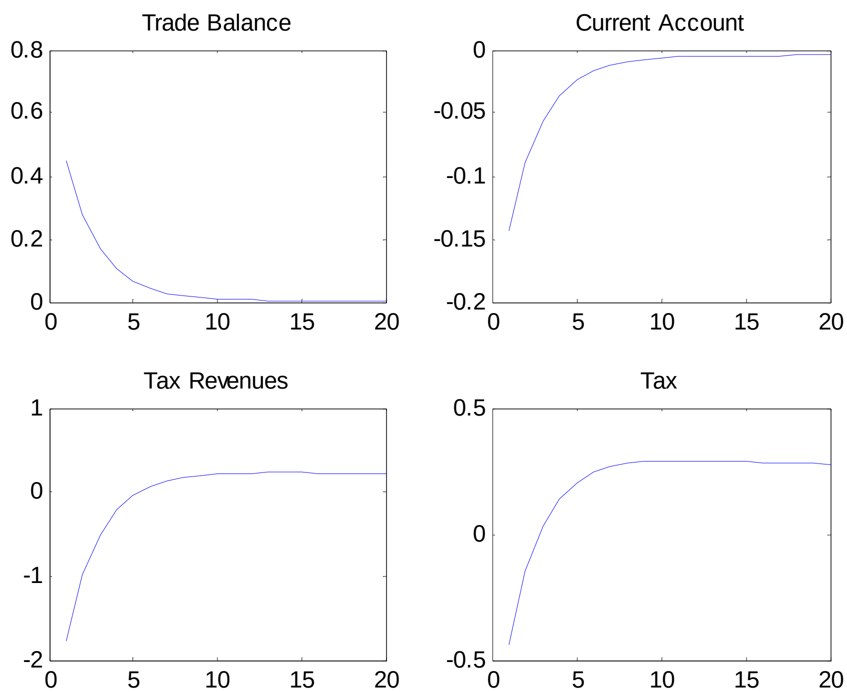

Figure 2: Impulse Responses to a Productivity Shock

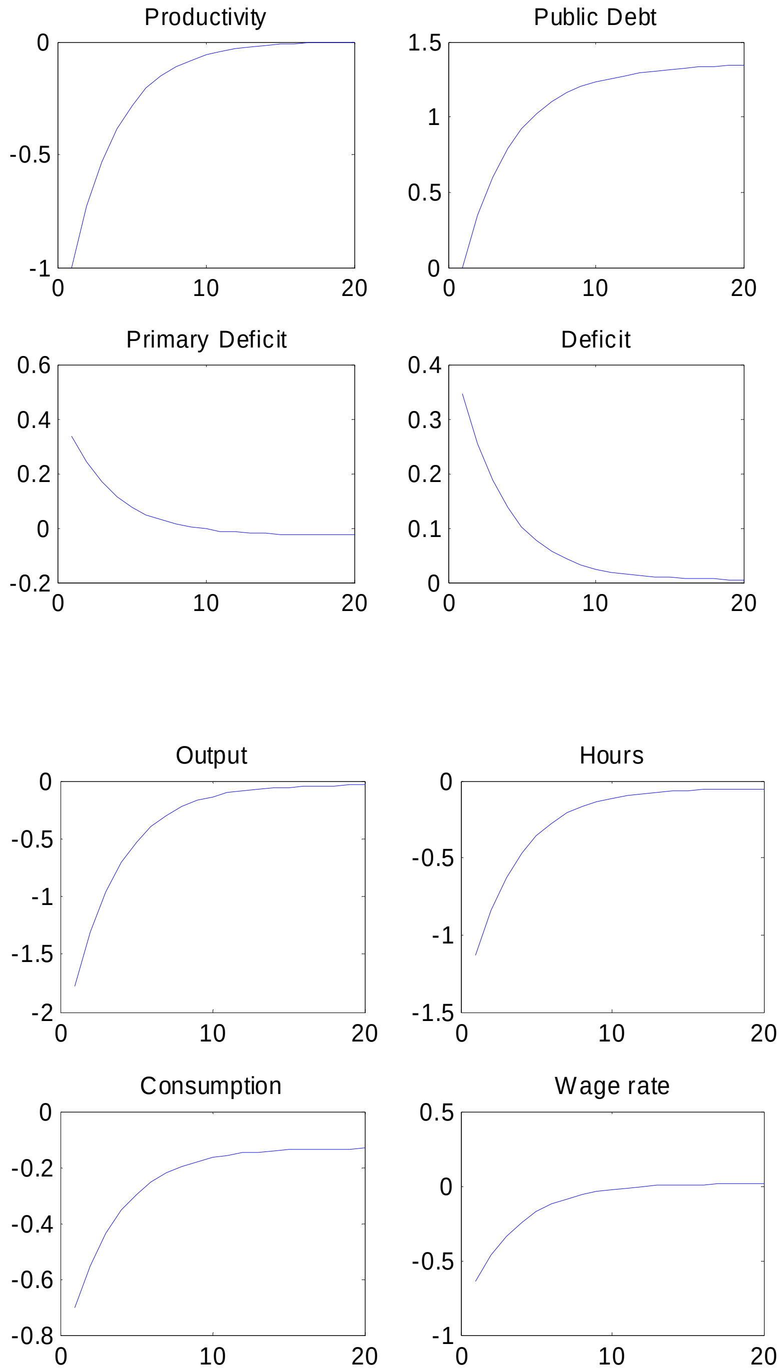

Figure 2: Impulse Responses to a Productivity Shock (cont)

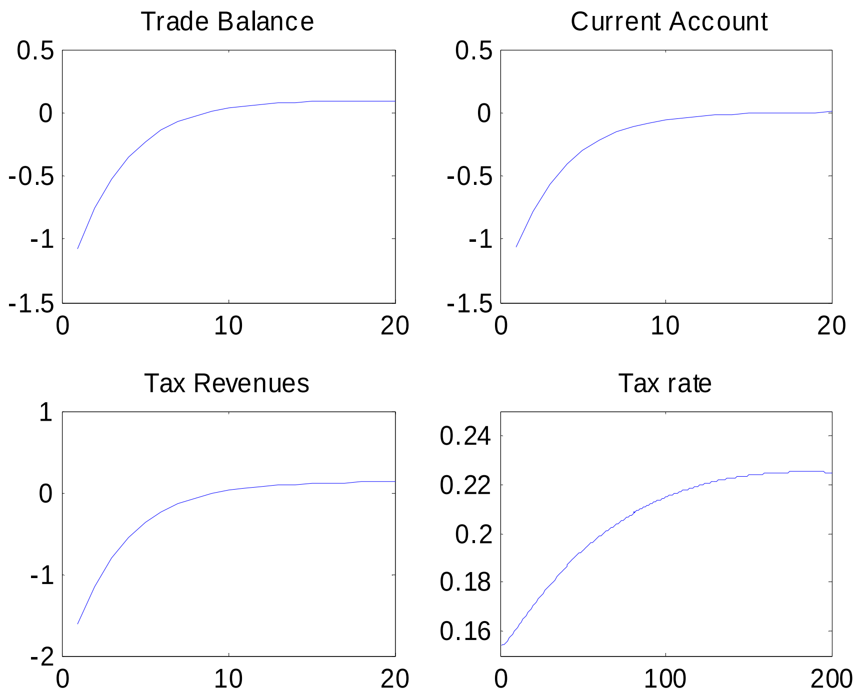

Figure 3: Impulses Responses to a Government Spending Shock

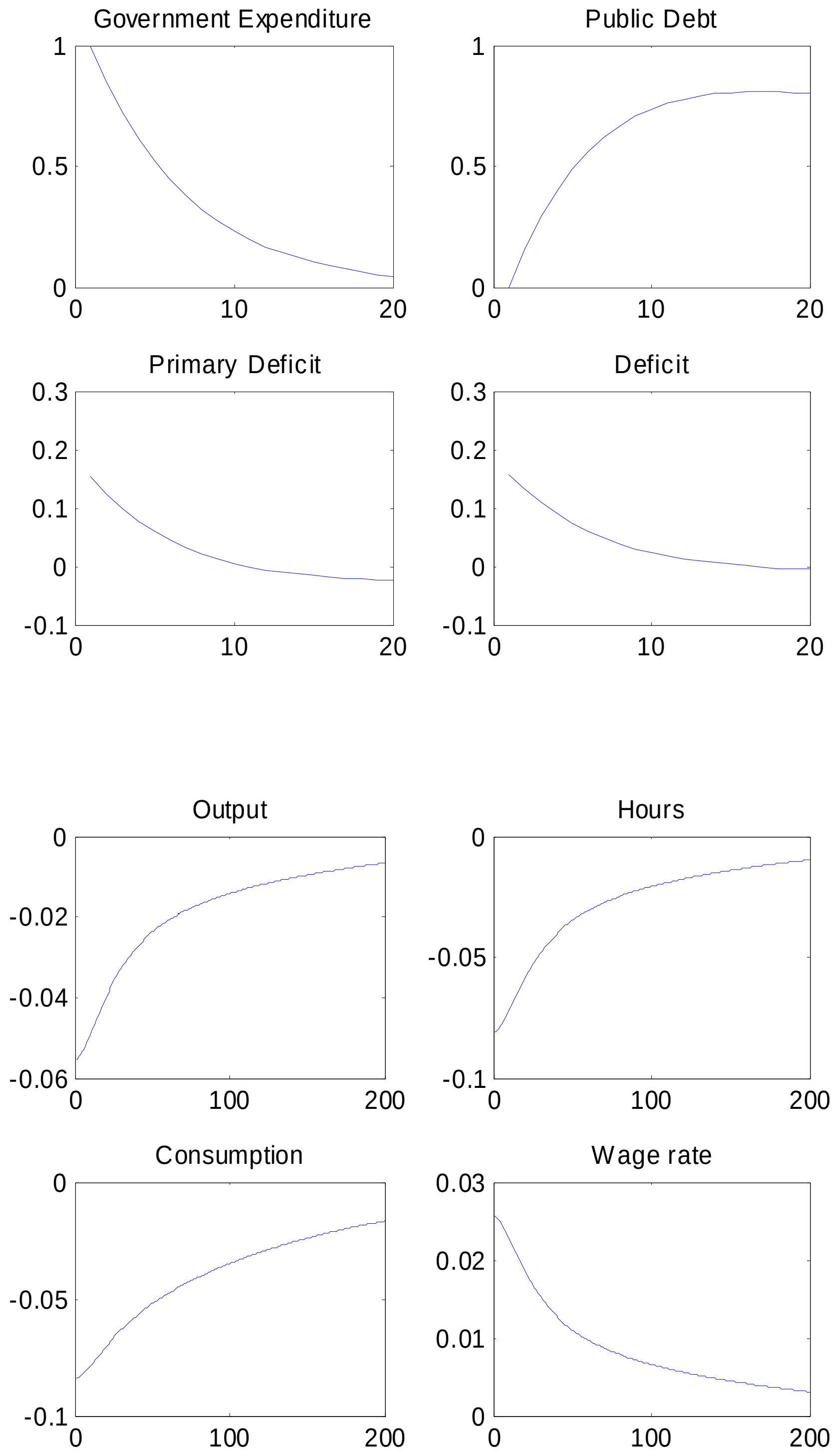

Figure 3: Impulse Responses to a Government Spending Shock (cont)

Figure 4: Impulses Responses to an Interest Rate Shock, $\nu = 1.1$

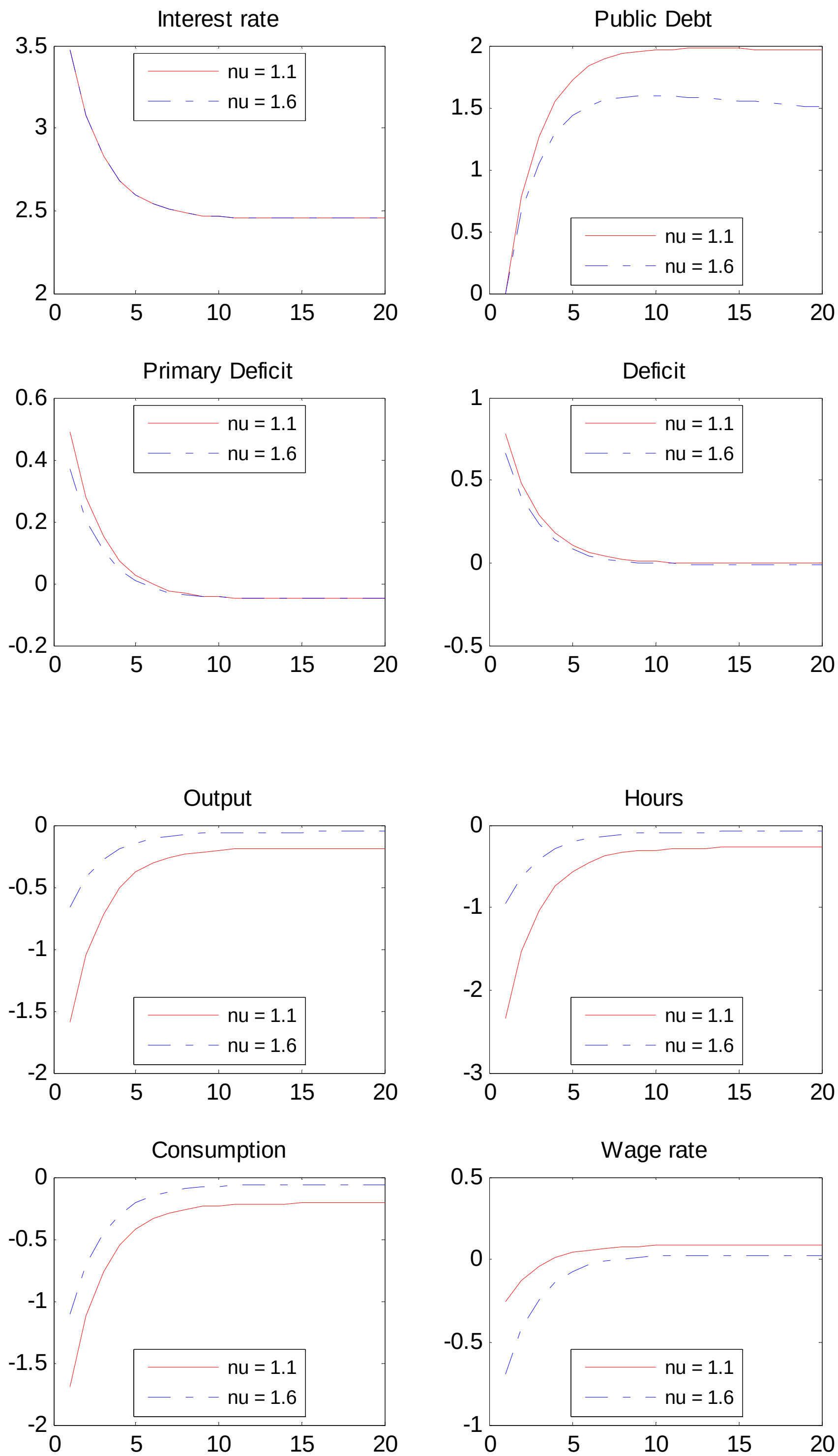

Figure 4: Impulses Responses to an Interest Rate Shock, $\nu = 1.1$ (cont)

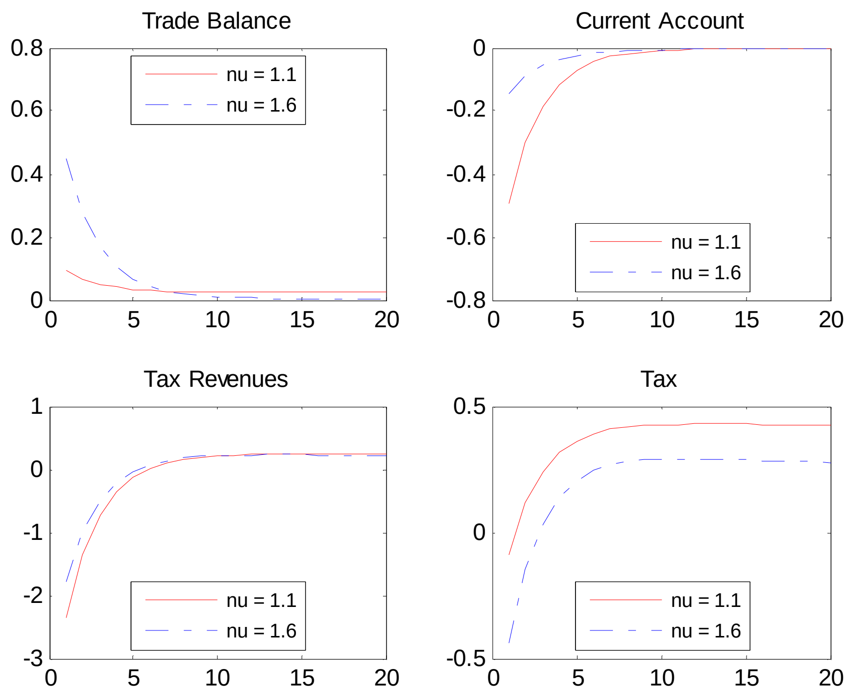

Figure 5: Impulses Responses to an Interest Rate Shock, $\theta = 0.5$

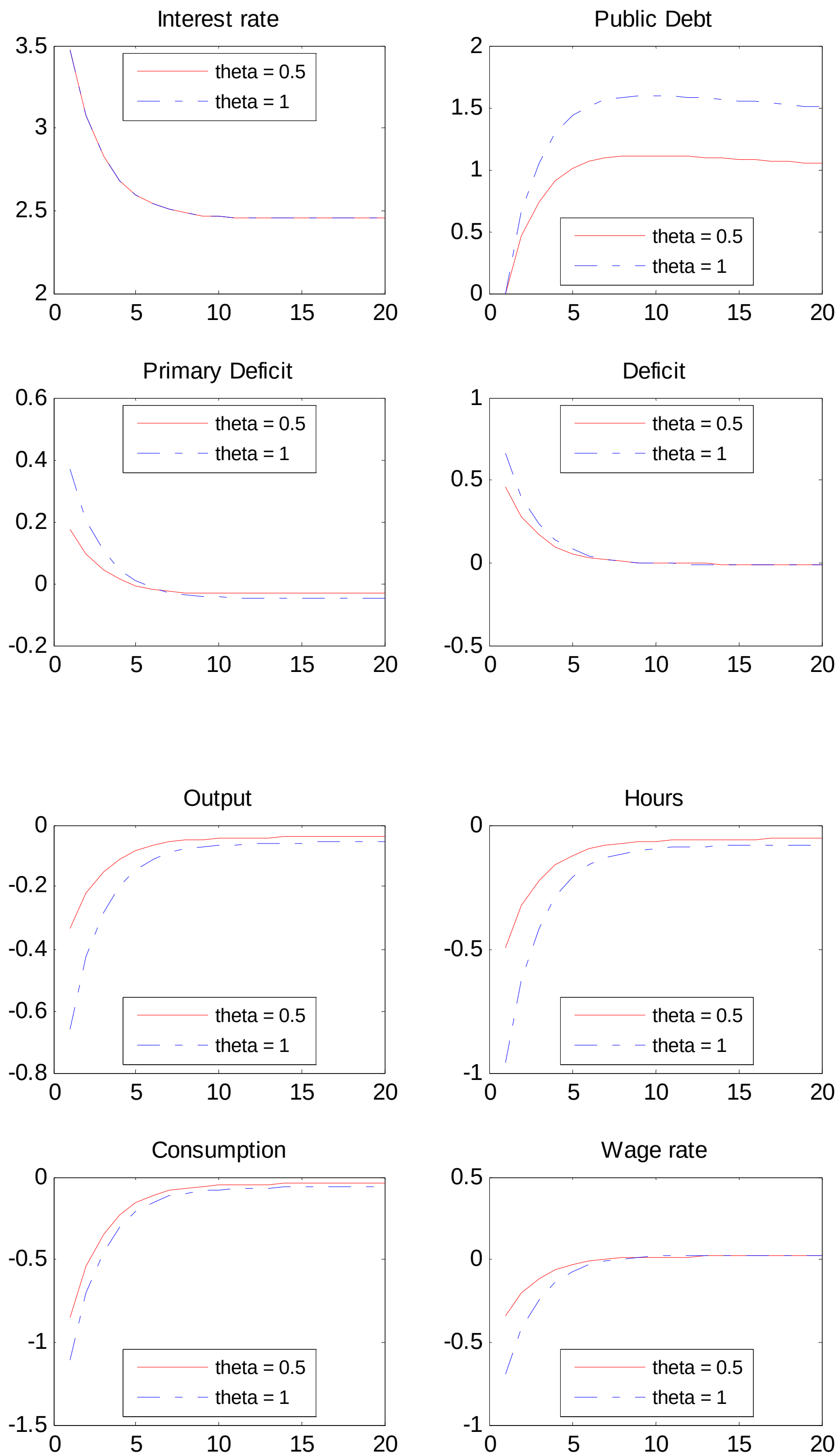

Figure 5: Impulses Responses to an Interest Rate Shock, $\theta = 0.5$ (cont)