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You, Me and the Mean: a Semiparametric Approach to the Redistributive Effects of Transfer Programs*

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Abstract: I examine how changes in the receipt of social transfers benefits associated to program reforms have affected the Canadian income distribution over the 1996-2006 period. Using the Survey of Labour and Income Dynamics, I apply nonparametric decomposition methods to construct density counterfactuals that identify the distributive effects of each transfer program. Counterfactual densities are constructed by reweighting the income distribution with propensity score functions that reflect changes in the probability of receiving program benefits. I find that reforms to the Social Assistance program have reduced its redistributive effectiveness, whereas Child Benefits, Employment Insurance and Old Age Security are more redistributive than in 1996.
Keywords: Redistribution, income inequality, tax and transfer system, nonparametric methods
JEL Classification: H23, D31, I38, C14

Resumen: Examino cómo cambios en la recepción de transferencias sociales asociados a las reformas de programas han afectado a la distribución de ingreso en Canadá en el periodo de 1996 a 2006. Utilizando la Encuesta de las Dinámicas del Empleo y el Ingreso en Canadá, aplico métodos no paramétricos de descomposición para construir densidades contrafácticas que identifican los efectos distributivos de cada programa de transferencias. Las densidades contrafácticas se construyen ponderando la distribución del ingreso por la función de emparejamiento por propensión, la cual refleja cambios en la probabilidad de recibir beneficios de los programas. Se encuentra que las reformas al Programa de Asistencia Social han reducido su eficacia redistributiva, mientras que el Programa de Apoyo Familiar a Hijos, el de Seguro de Desempleo y el Programa Público de Pensiones son más redistributivos que en 1996.
Palabras Clave: Redistribución, desigualdad del ingreso, sistema de impuestos y transferencias, métodos no paramétricos

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1 Introduction

In most developed economies the tax and transfer system significantly reduces market income inequality. However, the equalizing role of the tax and transfer system seems to be declining. There is bemusing evidence indicating that inequality of after tax and transfers income (disposable income) has grown faster than the inequality of market income in the 1990s and over recent years.[1] In Canada, for instance, the average annual growth of market income inequality between 1993 and 2008 is 0.12%, whereas the inequality of income after transfers rose four times faster, at 0.48% growth rate.[2] This evidence suggests the transfer system has slowed down its progressivity.

Transfer programs have the potential to affect income disparities and the distribution of income. Starting in the mid-1990s in Canada, changes in four sizable social programs that account for 45% of total government transfers occurred due to changes in the legislation or in the demographics considered in program elegibility criteria.[3] This paper addresses the question of whether the receipt of program benefits reported by families changed with program reforms enacted between 1996 and 2006, and how these changes influenced the distribution of after-transfer income. The results indicate that significant changes in the recipiency of benefits occurred and that these had important redistributive effects. Further, changes in the recipiency of each program affected different income regions along the distribution. Social Assistance has become less redistributive at the bottom of the distribution, whereas Child Benefits and Employment Insurance have become more redistributive up to the median income families.

Assessing the effects of policy changes on income distribution is crucial for governments

[1] Immervoll and Richardson (2011) reports that in OECD countries reporting full tax information, inequality of disposable income rose more quickly in the 1990s, although the rise in market income inequality slowed significantly. These countries are Australia, Canada, Denmark, Finland, West Germany, Israel, Netherlands, Norway, Sweden, Switzerland, United Kingdom, United States.

[2] Author's estimations based in the Survey of Labour and Income Dynamics, 1993-2008. Inequality is measured in terms of income in real family equivalent units, using the Gini coefficient.

[3] The programs under examination are Social Assistance (or Welfare) which became less generous and more restrictive after the reforms; Child Benefits System, which became more accessible and more generous, Employment Insurance, which became more restrictive; and Old Age Security whose legislation did not change but that saw an increase in the proportion of eleigible individuals as a result of demographic changes.

concerned about inequality. Here, I use the Canadian longitudinal Survey of Labour and Income Dynamics (SLID) which includes not only standard demographic and labour information, but also data on taxes and transfers reported directly in the tax files for over 80% of the repondents who granted consent to Statistics Canada. For the analysis, I employ nonparametric methods to investigate the effect of changes in Canadian social programs on the income distribution. Specifically, I look at the differences in after-transfer income densities between 1996 and 2006, using the nonparametric reweighting decomposition method proposed by DiNardo et al. (1996). This approach identifies changes in the income distribution derived from each transfer program through the construction of counterfactual densities. The counterfactual analysis describes how the density of income would have been in 2006, had the recipiency of a given benefit remained as it was in 1996, before it was reformed. The counterfactual density is an adjusted version of the density in 2006 that weights income with a ratio reflecting the change in the probability of receiving program benefits before and after the program reform. This *weighting* factor identifies how the likelihood of receiving program benefits changed for comparable families given a set of observed characteristics including program eligibility requirements – propensity score function.[4] As a result, the effect of changes in legislation to programs are captured through the counterfactual density based on the observed receipt of program benefits reported by families.

There are significant advantages to the use of nonparametric methods. The more obvious is the abundance of available statistics to measure changes in the distribution. Nonparametric methods fully characterize the income distribution and allow to look at the effects of programs over the entire distribution. This approach is best suited to identify the region of the income distribution where most of the changes occur, especially when the effects are locally concentrated around a certain income level. In addition, because a counterfactual is constructed for each program, this approach enables to separate out the effect of changes to multiple programs occurring during the same period and to describe the relative contribution of each program in explaining changes in the income distribution. Previous studies document dis-

[4]Province, number of young children, age, sex, marital status, experience, and program specific characteristics reflecting eligibility criteria are examined.

tributional changes using scalar descriptive statistics such as the Gini coefficient, percentile ratios and other inequality measures (Saez and Piketty,2007; and Frenette et al., 2009). These distributional statistics may not identify *where* in the distribution of income program changes have larger effects.[5]

The underlying assumption of this work is that families' response to changes in programs are summarized by the before and after reform ratio of probability of program receipt. In practice, program reforms may increase their effectiveness in reducing inequality by either changing the intensive (generosity) or extensive margin (eligibility) of the benefit. Augmenting generosity will rise income at the bottom of the distribution by either increasing the amount of benefits a given group receives; while expanding elegibility requirements will decrease the number of families receiving a given amount of benefits.[6] I analyze only changes in the recipiency of benefits without specifically account for changes in the generosity or the eligibility of the programs.

The main contribution of the paper is the application of nonparametric decomposition techniques commonly used in labor economics to analyze the distributive effects of changes to transfer programs. This approach yields more detailed description of the distributive effects of social programs, and it is especially advantageous when program changes affect the distribution locally around a specific income level. To my knowledge this is the first study where these methods have been applied in public finance. As crucial as it is, the role played by changes to the tax and transfer system on the income distribution has been scarcely examined using individual data. This is primary due to the lack of microdata on tax and transfers together with information on the program eligibility requirements for respondents. Some studies have used administrative data with limited information of taxpayers' socioeconomic characteristics, while others use census or household surveys with little information on in-

[5]Frenette et al. (2009) and Lemieux (2010) discuss advantages of nonparametric methods to examine distributional changes.

[6]Theoretically, increasing the generosity of a program (or facilitating the services of a program) reduce the stigma associated with receiving benefits and increase recipiency (take-up) of benefits, while spending cutoffs tighten competition among eligible families applying for benefits, reducing benefit receipt. Discrepancies between eligibility and take-up of benefits are examined in Moffitt (2002), Currie (2004), and Kleven and Kopczuk (2011).

come and taxes. A third approach tries to get around these data limitations using imputation techniques to link *estimates* of tax information from administrative data with census' socioeconomic information (See Frenette et al., 2009 for Canada, and Saez and Piketty,2007 for the U.S.) There are, however, several donwnsides of this approach that limit the interpretation of the estimated effects.[7]

As Frenette et al. (2009), I find that the transfer system became less effective in reducing income inequality. However, the effect of specific programs is diverse. In sum, I find that Child Benefits reduces the share of low-income families but increases income disparities between the percentiles. Changes in the receipt of Old Age Security benefits, although are not preceded by legislative changes, show modest distributive effect reducing low-income families and rising the cluster of medium income families. Employment Insurance reduces inequality mostly at the low half of the distribution, whereas changes in the receipt of Social Assistance significantly increase income disparities by rising the share of low-income families relative to 1996. Changes in sociodemographics, and in Child Benefits explain most of the changes at the top 90 and 95 percentiles. Although the evidence is based on Canadian data, the lessons on the redistributive effects of transfer programs are relevant for other economies.

The empirical approach has some drawbacks. Decomposition models do not recover behavioral relationship or 'structural' parameters between income and the factors (transfer programs); however, they indicate which factors are quantitatively important and consequently, point out what hypothesis are there to be explored. The analysis is not able to clearly identify a causal impact of each program or changes in the program parameters on the income distribution. The model does not account for general equilibrium or dynamic effects, for instance, the effects of program reforms on taxpayers are not explicitly examined. Labour supply changes induced by program changes are also not followed.

[7]The first disadvantage is that the simulated response to policy changes is *exogenous* to the actual behavior of families. The effects are assumed to be exogenous to any behavioral response and are based on simulations that assume full tax compliance, and 100% program take-up rates – target population is fully reached by the program. The second is that the imputation techniques imply a degree of measurement error that is carried to the estimation of policy changes, which may obscure the actual effects. This is particulary relevant when the policy effects are linearly estimated and the standard estimation assumptions may not hold.

What remains in the paper is organized as follows: the next section describes the observed changes in the income distribution, the reforms to the legislation and the changes of programs receipt reported by Canadian families. Section three presents the empirical strategy and its identification assumptions required to clear out the effect of changes in the receipt of programs benefits on changes in the distribution. Section four describes the data and income density estimators for families receiving program benefits in 1996 and 2006. Results are discussed in section five, while a modified version of the model examining differences in the distribution of disposable income is discussed in section six. Section 6 concludes the paper.

2 A decade of changes: 1996 - 2006

2.1 Changes in the income distribution

The increase in market income inequality in Canada and in other nations in the last three decades has been documented in numerous studies. In Canada, the top 20% of income earners in 1980 perceived 44% of total earnings, while by 2007 the top 20% earned 51%. Frenette et al. (2009) and Fortin et al. (2012) document how the Canadian tax and transfer system effectively reduced a great part of market income inequality in the 80s; and how, since the mid-90s, inequality of disposable income started increasing and the tax and transfer system became less equalizing. What seems to be more concerning about recent trends in inequality is its persistence (Bénabou, 1996). Fortin et al. (2012) describe that inequality in Canada grew sharply during recessions of 81-83 and early 90's, but it did not decline as much during economic booms.

Figure 1 shows trends in Canadian inequality for various income measures, using the growth in the associated Gini coefficients (relative to 1993). Inequality of market income (dotted line) has remained fairly stable over the 1993-2008 period, as suggested by the flat growth of the associated Gini. On the other hand, the Gini associated with both, after transfers income inequality (slashed line) and disposable income inequality (solid line) has rapidly increased over the period. The role of the tax system is also shown in figure 1, which depicts the equalizing index, measured as the rate of growth in the difference between disposable and market income, relative to 1993.[8] The equalizing index rapidly declined after the mid-90s, a period that viewed major reforms to social programs, and stabilized during the later years of the sample. The evidence from figure 1 suggests that the tax and transfer system became less equalizing between 1993 and 2010.

Assessments of changes in the income distribution based on single-valued descriptives like the Gini coefficient are limited. These summary measures of inequality fail to indicate the region of the income distribution in which most of the changes occur (Lemieux, 2010). In addition, they might be bad indicators of the effectiveness of social programs. Measures like the Gini coefficient put more weight to changes occurring in the middle of the income distribution rather than on the tails, hence missing reforms induced by social programs that are likely to have a bigger effect on the tails of the distribution (Atkinson, 1970).

A full characterization of the after-transfers income distribution (using density kernel estimators) is provided in figure 2 for the years 1996 and 2006.[9] The graph on the left shows the after-transfers income distribution for the two years, with the 2006 distribution lying to the right of the 1996 distribution. The graph on the right shows the difference between the two distributions. In 1996, there was a greater mass of low income families up to the point where the two densities cross relative to 2006. The differences in 2006 relative to 1996 are reported in the graph on the right. For any given level of income up to 50 thousand, there is less concentration families in 2006 than 1996, and a greater mass of families with income above this level. The figure indicates that there has been a reduction in the concentration of low-income families, but also an increase in the disparity of income.[10]

[8]The equalizing index is defined in terms of the Gini coefficients: $\left({}^{Gini(y_{Disposable})}/_{Gini(y_{Market})} - 1\right) \times 100$. An equalizing index less than 1 indicates that the role of the tax and transfer system is declining relative to 1993

[9]The income definition used in the density estimation is expressed in real adult equivalent dollars of 2010, which refers to total family income scaled by the size and composition of the family.

[10]Similar results are shown by Lu et al. (2011) and Fortin and Schirle (2006) for Canada, Daly and Valetta (2006) for the U.S. and Hyslop and Maré (2005) in New Zealand.

2.2 Program reforms

In Canada, personal income tax and transfer programs have changed substantially over the past 15 years. The focus of this study is on personal income transfers programs whose legislation changed starting the mid-90s. Program benefits or transfers are defined as direct payments from the government to eligible individuals or families. This section describes the four selected transfer programs and their reforms.

For the purpose of this study, I select four influential programs which significantly changed their legislation during the period of study (1996-2006) or the demographics linked to their elegibility requirements. The economic significance of these programs is substantial as they jointly account for 22% of total revenue. These include the Canada Child Tax Benefit System (CB), Old Age Security (OAS) including the Guaranteed Income Security, Social Assistance (SA) and Employment Insurance (EI).[11]

Child Benefits

The Canada Child Tax Benefit system (CB) is a set of non-taxable programs that provide allowance to families with children. Milligan and Stabile (2007) document a summary of CB reforms. Until 1998, benefits were assigned irrespective of income; since then, families with income above a threshold are required to repay part of their benefits. Also in 1998, a major reform occurred to the CB system with the introduction of the National Child Benefit Program (NCBP) and the addition of a new Supplement for each young children. The Child Tax benefit program was combined with the Working Income Supplement into a denominated Canada Child Tax Benefit.[12] Eligibility depends on having young dependent children, and

[11] Two thirds of total cash transfers are financed out of general revenues, and the remaining part out of payroll taxes. The three largest programs paid of general revenues changed during 1996-2006 are SA, CB and OAS. Nonetheless is financed out of payroll taxes I also examine EI because its benefits are taken up only during contingent jobless periods and because its legislation was importantly reformed.

[12] The Canada Child Tax Benefit (CCTB) monthly transfers non-taxable income to assist families raising children aged up to 17 years. It includes additional financial assistance for the care of children with severe or prolonged mental or physical impairments under the Child Disability Benefits Program. It also provides cash transfers for low-income families with children under the National Child Benefit Supplement. In 2006, the Child tax benefit spent 9.4 billion dollars, the equivalent to 6.5% of all (federal and provincial) government transfers. In some provinces, child benefits may be clawed back from social assistance payments, as documented in Milligan and Stabile (2007)

the benefits depend on the income level and number of children. The maximum benefits for one-child families was $1,625, for two-child families $3,050 and $1,425 dollars for each additional child. Benefits get reduced for families with net income above $25,921 dollars. The percentage of families receiving CB declined 5.6% from 1996 to 2006.

Figure 3 shows in a solid line the after-trasfer income densities of families in 1996 (top graph) and in 2006 (bottom graph). In each graph the overall distribution is decomposed into the distribution of families receiving a benefit (shadowed distribution) and those not receiving a benefit (dotted line). Comparison of these two distributions provides an approximate idea of benefit recipiency rates. The two graphs in the first column of the figure refer to child benefits. In 1996, similar fractions of poor families (income under $38,000) received CB and virtually no family over $50,000 received this benefit in 1996. The mass of families at the bottom of the distribution declined in 2006, and families with higher income levels increased. This might be due to changes in the program mentioned above or due to demographic changes (in fertility or family composition) between the two years.

Old Age Security

The Old Age Security (OAS) provides pension to the elderly aged sixty five. Although it is a universal pension plan, seniors with annual income exceeding $53,215 of 1996 repay 15% of the amount exceeding the pension base. As a result, high-income pensioners do not receive OAS benefits (Strick, 1999). In 1967, the Guaranteed Income Supplement was introduced as an income test supplement for seniors with income needs receiving Old Age Security. Seniors older than 60 years would receive no-taxable benefits ranging from $11,420 for single seniors with no other income, and gets reduced $0.50 for every dollar of outside income and decreasing up to $350 for those with income exceeding $50,000. To give an idea of the magnitud of the program consider that in 2006, OAS delivered benefits to 18% families, twice the CB coverage, and spent 30 billion dollars, 3 times the spending of CB.[13]

The legislation of the OAS was not reformed during the period of study. There are however two reasons to examine changes in OAS take-up. First, OAS is the program financed out

[13] Percentage of families receiving benefits are the author's calculations based on the SLID.

of general revenues with the greatest spending: 3.6 million seniors took OAS benefits up in 1996 for $16 billion dollars; in 2006, 4.3 million seniors received $30.5 billions.[14] Second, there are significant changes in the take-up of OAS benefits between 1996 and 2006 derived from the increasing proportion of seniors that reflect the aging of the population, which increased by 11% relative to the 1996 level.

The second column of figure 3 shows the income densities of families with and without OAS benefits in 1996 and 2006. The figure documents the increase in the fraction of families with income ranging 30 to 70 thousand receiving OAS between the two years.

Social Assistance

The Social Assistance program is administered by provincial governments mandated to provide income benefits to those unable to work for long periods including single-parent families with young children, the disabled, and those with physical health conditions who are permanent unemployable, and the not yet pensioner senior. Eligibility is related to assets, income and a test of needs. SA benefits vary across provinces. A major reform occurred in 1996, when the federal government replaced federal transfers payments to the provinces under the Canada Assistance Plan (CAP) with a bundle of mixed resources for health, education and social assistance under the Canada Health and Social Transfer (CHST). Provinces started changing the criteria for recipients and conditioned benefits to job searching, and employment and reach those in more need such as lone parents with young children and people with disabilities. Kneebone and White (2009) documents provincial changes. Whelan (2010) documents take-up rate of Social Assistance in Canada and compares it to other countries. Whelan's SA take-up rate estimate for 1997 is 32% when only income is used as eligibility criteria, and 47% when he uses information on both income and asset criteria.

Relative to other programs, SA's coverage of families is small and those who take up benefits concentrate at the low-tail of the distribution. The decline in the concentration of families receiving SA benefits from 1996 to 2006 is approximately 50% (third column of figure 3).

[14]Data on spending for all programs is from CANSIM table 385-0009, and form the *2011 CPP and OAS Statistics Book* at http://www.servicecanada.gc.ca/eng/isp/statistics/cppstatbook/statbook2011.

Employment Insurance

The Employment Insurance program (Unemployment Insurance until 1996) provides unemployment benefits, sickness benefits, maternity and parental benefits, approved training course, and job-creation or job-sharing projects. Eligibility is conditional on having worked a minimum number of hours, and both, employees and employers pay in-advance premiums. Benefits are assigned temporarily to whom reports jobless periods and fulfills the eligibility requirements.

Motivated by previous empirical studies questioning the generosity and moral hazard problems, various reforms to EI were launched in 1996. Requirements became more strict (increases in hours of work) and penalties for workers with recurrent claims were introduced, although these penalties no longer exist. Incentives for unemployed workers to accept part-time jobs while seeking full-time employment were designed to reduce some of the identified moral hazard problems. EI is another sizable social program, with benefits paid amounting to $12.5 billions in 2006.

Overall, the proportion of families with EI take-up fell from 24% in 1996 to 19% in 2006. The income densities of families with EI take-up are shown in the forth column in figure 3. Families receiving EI benefits report a wider income range in 2006, with a modal income level of $40,000.

The reforms to the four selected programs are summarized in table 1.

2.3 Program interactions

Unlike previous studies, I document the possibility that individuals receive benefits from multiple programs, which I refer to as *program interactions*. Program interactions are reported in the second panel of table 2. The most common combination of benefits is that of CB and EI, followed by that of CB and SA. The percentage of couples receiving any combination of program benefits dropped between 1996 and 2006 with the solely exception of those receiving CB and OAS. The SA program used to support families with children, and since 1998, benefits to children have largely been displaced by the Child Benefit System.[15] Note that programs *interacted* with SA report the highest percentage declines. In the same token, the rise in OAS receipiency dominates its interactions with other programs, in particular with CB which increased 42% relative to the 1996.

Given the relative small proportion of families receiving two or more programs, estimates accounting for program interactions are not expected to differ greatly; yet, understanding the incentives to take up benefits from multiple programs may be relevant in estimating the redistributive effects of programs as they can magnify or reduce each program's effects.

3 Empirical framework

I apply DiNardo et al. (1996) nonparametric decomposition methods to study changes in the income distribution in Canada between 1996 and 2006. Rather than using aggregate mean and dispersion measures, nonparametric methods use kernel density estimators that describe the full distribution of income. Further, it allows for flexible functional forms to fully account for the heterogeneous effects of policy on the population.

The construction of counterfactuals is a crucial identifying piece in this analysis. The type of counterfactual I propose describes how the *density* of income would have been in 2006 if benefits from a given program remained being delivered as in 1996. Table 2 shows the changes in benefit recipiency over the period for each program. Recipiency diminished for all programs over the period, except for OAS. Overall recipiency of Child Benefits declined by 5.6% between 1993 and 2009. The fraction of families receiving this benefit declined during the economic boom at the end of the 1990s and rose again after 2003. OAS recipiency is the only benefit that has increased recipiency (by 12.3%). This is a consequence of the pace of the aging population, as life expectancy rates continue rising and a larger stream of baby-boomers reaches retirement age. SA recipiency also declined during the period by 47%,

[15]Kneebone and White (2009) describes provincial variation in the EI and SA program interaction. For instance, tighter EI eligibility rules in provinces with relatively low unemployment makes it easier for people with jobless spells to take up SA benefits, relative to residents of provinces with trends of high unemployment rates. Milligan and Stabile (2007) also examines the integration of CB and SA.

as did EI by 21%. Receipt of EI was high during the long period of economic downturn in the first part of the 1990s and declined during economic recovery.

3.1 Empirical design of policy's distributive effects

Evaluating the effects of policy changes is a central activity of economics. However, its estimation often requires demanding data sets and models that make this a daunting task. In this application, I plan to decompose changes in family income between 1996 and 2006 into changes due to differences in income recipiency resulting from changes to several income support programs. This exercise allows to isolate the distributive effect of changes to different government programs that took place in the late 1990s and the beginning to 2000s and quantify the contribution of each change to overall Canadian inequality. Market income is assumed exogenous as market income inequality remains fairly constant over the period considered.

Let $j \in J$ be a set of policies, which in this paper will refer exclusively to transfer programs, and $t = \{1996, 2006\}$ where $t = 1996$ denotes the pre-reform date and $t = 2006$ the post-reform date. We assume that an individual i at time t has income y_{it} generated by some income-generating function $g(\cdot)$. Income is determined by this income-generating function depending on the receipt of government transfers from j programs, observable characteristics X_{it}, such as age, sex and education and a set of unobservable characteristics U_{it} likely affecting person's i income. Hence:

$$y_{it} = g_t(X_{it}, \tau_{ijt}, U_{it})$$

where g_t is the income generating function that can vary over time and τ_{ijt} is a binary variable indicating whether person i receives benefits of the j-program at date t.

According to the expression above, there are two main sources of variation that can explain income density differences between the pre-reform date and the post-reform date. One comes from changes in the distribution of either of the determinants of income (X, τ, U) and the second comes from changes in the structure parameters underlying the income-generating function g. As explained below in more detail, the decomposition thechnique employed here separates these two sources of change, by introducing counterfactual densities. The counterfactual density isolates the effect of each determinant of income on the distribution of income and allows to separate the effect of changes in the distribution of characteristics from changes in structural parameters.

The following distributive assumptions are neccessary to construct the counterfactual densities. They are required to eliminate confounding effects that would change the interpretation of the estimates. A more detailed discussion can be found in Fortin et al. (2011).

Identifying assumptions

Assumption 1 –*The ignorability condition*: The distribution of unobservables is time-independent. This is also called the *unconfoundedness* condition and it ensures that there are not other time-varying factors that determine income other than changes in the policy and in observed characteristics (X). In the nonparametric case, this condition requires that unobserved factors (U) are equally distributed over time; and it is analogue to the zero mean condition in the linear parametric case. This assumption allows for selection bias based on unobservables, as long as the unobservables are equally distributed over time; or more precisely, the model is well-identified as long as the *structural* relationship between the 'outcome' and 'unobservable' is the same in 1996 and in 2006.[16] The ignorability condition would fail to hold if the *distribution* of unobserved characteristics, such as income tax, changes over time. To the extent to which this condition holds the redistributive effects in the model would be unbiased.[17]

Assumption 2 – *The overlapping support*: The probability of receiving program transfers conditional on observables (X) is in the interior set $(0, 1)$. This condition eliminates the possibility that nobody in the economy receives program benefits in a given year (non existent

[16]It is common to assume linear parameters and additive separability of the outcome-state functions between observable and unobservable components (Oaxaca, 1973). In this case, unobserved characteristics are assumed to be (mean) independent of X with zero mean $E[\epsilon_t|X_t]$.

[17]I checked whether the results from decomposing disposable (after-tax) income vary relative to results from decomposing after-transfers income. Factor relative contribution in explaining changes in the distribution is robust and results are discussed in section 5.3.

program before or after). It ensures that the reweighting factor used to construct the counterfactual is well defined, ruling out degenerated or nonexistent propensity score functions. This assumption would fail to hold, for instance, if a program is eliminated or the take-up rate is zero because nobody is eligible.

Assumption 3 –*Income structure function*: The structure function of income, represented by the conditional income density of income on observable characteristics (τ, X) is independent of the distribution of the observables. This condition isolates the effect of the policy from possible changes driven by the way the economy is determining income, which in turn enables the construction of the counterfactual densities.[18] This requirement of decomposition methods rules out the possibility that changes in the distribution of characteristics (for instance crowding out of single mothers) to affect the returns to structural determinants of income.

Assumption 4 –*Self-selection based on observables*: The choice of whether to apply and be eligible to receive program benefits depends uniquely on observables (X) and not on unobservable characteristics (U). This assumption rules out confounding effects related to time-specific determinants of the decision to apply for program benefits that might be picked up by examining changes in the income distribution. It is sufficient to assume that time-invariant distribution of unobservables factors determining family's decisions to apply for program benefits. To the extent that unobservables such as social stigma of taking up social benefits is varies with the program reform would introduce bias on the estimates. Selection on unobservables is not addressed in this study.

Advantages and limitations of the model

This study does not account for general equilibrium or dynamic effects. I am not examining what are the effects of program reforms on taxpayers, or how the benefits received affect all members of the society. It is a static analysis as people's behaviour is not explicitly examined and responses to labour supply changes induced by program changes are not

[18]In the hypothetical case where individual income is linearly determined in terms of observed independent factors; this assumption would make the parameters $\beta_{X,96} = \beta_{X,06}$.

followed. In particular, my estimates do not account for labor supply effects.[19] Decomposition models do not recover behavioral relationship or 'structural' parameters between income and the factors (transfer programs); however, it indicates which factors are quantitatively important and consequently, it points out what hypothesis are there to be explored. Moreover, although I control for a set of sociodemographic characteristics, selection issues regarding family decisions to apply for program benefits are not taken into account. To the extent that individuals self-select to apply for program benefits based on unobservable characteristics, this may introduce some bias in the estimated distributive effects.

One advantage of this model is that reweighting nonparametric decomposition methods are best suited to identify the region of the income distribution where most of the changes occur, especially when the effects are locally concentrated around a certain income level. In addition, because a counterfactual is constructed for each program, this approach enables to separate out the effect of changes to multiple programs occurring during the same period. Analysis of the receipt of benefits from multiple programs can be addressed too. Finally, and in a more technical vain, there are three worth-noting advantages of nonparametrically estimating the effects of a policy. $i)$ the identification of the income-state function is not required; $ii)$ under the ignorability condition, the correlation between observables X and unobservables U is irrelevant, and $iii)$ if the ignorability and overlapping support conditions hold, this method produces consistent estimates of both the income-structure and the composition effects.[20]

3.2 The simplest model with one policy

Consider that each income observation comes from the joint distribution of income and individual characteristics (including program benefits receipt) and dates given by $f_t(y, X, \tau, U)$, where τ is a vector of binary indicators of the receipt of transfer programs $j \in J$.

In order to illustrate the intuition of the model without penalizing clarity of ideas with

[19]Keane (2011) surveys the literature on the responsiveness of labor supply to disposable wages and transfers, and concludes that labour supply elasticities for women are larger than for men especially in the long-run, once fertility, marriage, education and work experience are examined.

[20]Fortin et al. (2011) further outline the advantages of decomposition methods.

multiple dimensions, I here present the simplest version of the model in which income only depends of a unique transfer program τ. Since I am interested in isolating the effect of the policy changes between 1996 and 2006 (denoted Δ_τ), in what follows the period group will refer to these years. The joint density of income and of program benefits at date t, $f_t(y_t, \tau_t)$, can be expressed as the distribution of income conditional on the policy, $f_t(y \mid \tau)$, and the marginal density of the observed characteristics at date t, $f_\tau(\tau_t)$. Furthermore, the unconditional density of income at date t, $f_t(y_t)$, can be expressed as follows:[21]

$$f_{06}(y_{06}; \tau_{06}) = \int f_{06}(y \mid \tau) dF_\tau(\tau_{06}) \tag{1}$$

The goal is decomposing the overall difference in the income distribution denoted by Δ_O:

$$\Delta_O = f_{06}(y_{06}, \tau_{06}) - f_{96}(y_{96}, \tau_{96}) \tag{2}$$

Expression (2) can also be written as:

$$[\Delta_O \mid \Delta\tau] = \underbrace{\underbrace{f_{06}(y_{06}; \tau_{06})}_{\text{Observed}} - \underbrace{f_{06}^{C_\tau}(y_{06}; \tau_{96})}_{\text{Unobserved}}}_{\text{Composition effect } (\Delta_\tau)} + \underbrace{\underbrace{f_{06}^{C_\tau}(y_{06}; \tau_{96})}_{\text{Unobserved}} - \underbrace{f_{96}(y_{96}; \tau_{96})}_{\text{Observed}}}_{\text{Structure effect } (\Delta_S)} \tag{3}$$

where $f_{06}^{C_\tau}(y_{06}; \tau_{96})$ denotes the counterfactual density, defined as:

$$f_{06}^{C_\tau}(y_{06}; \tau_{96}) = \int f_{06}(y \mid \tau) dF_\tau(\tau_{96}) \tag{4}$$

Expression (3) is generated by adding and subtracting a hypothetical density with intuitive meaning: *the density of income that would have been in 2006, if the transfer program had remained as in 1996, without the reform.* The counterfactual density serves to identify the effect of policy changes by separating the composition (Δ_τ) and structure effects (Δ_S). The composition effect describes the *"explained"* differences in the distribution of characteristics τ between period groups. The structure effect refers to differences in the parameters of the

[21]The unconditional density of income at date t can be denoted indistinctly as $f_t(y_t)$ or as $f_t(y_t; \tau_t)$. The semicolon serves to fully express the remaining components of the joint density as part of an useful notation for defining the counterfactual densities.

income generating function g over time.

Given that $dF_\tau(\tau_{96})$ in expression (4) is not observable, the key to construct the counterfactual density is to introduce a reweighting factor that transforms the marginal distribution into observable terms. This can be achieved by multiplying and dividing it by the marginal distribution of τ_{06}.

$$f_{06}^{C_\tau}(y_{06};\tau_{96}) = \int \psi(\tau) f_{06}(y \mid \tau) dF_\tau(\tau_{06}) \tag{5}$$

The assumption 3 is required to write the counterfactual density as in expression (5). In this sense, the counterfactual density can be understood as an adjusted version of the observed density in the 2006 group, where $\psi(\tau)$ is the adjustment (or *reweighting*) factor

$$\psi(\tau) = \frac{dF_\tau(\tau_{96})}{dF_\tau(\tau_{06})}$$

The reweighting factor reflects changes over time on the probability distribution of policy τ. When decomposition factors are binary variables such as whether the individual receives programs benefits, $\psi(\tau|X)$ can be rewritten as the propensity score function in terms of a vector of characteristics (X) that influence the probability of receiving benefits from program τ, as follows:

$$\psi(\tau \mid X) = \tau \frac{Pr(\tau_{96} = 1 \mid X_{96})}{Pr(\tau_{06} = 1 \mid X_{06})} + (1-\tau) \frac{Pr(\tau_{96} = 0 \mid X_{96})}{Pr(\tau_{06} = 0 \mid X_{06})} \tag{6}$$

The intuition of the reweigthing function is that it reflects the change in the likelihood of receiving program benefits due to the program reforms, which can be estimated with a logit or probit model, once other observed family characteristics are kept constant over time. The propensity score reweighting function describes changes over time for individuals who have identical observable characteristics after being paired across year groups. The vector X contains variables reflecting program requirements, such as binary for sex, married/common law, living with dependent children under 17 years old, province, full-time employment, sex interacted with education and continuous variable for experience and quartic of experience.

Family responses may in consequence, be sufficiently large or sufficiently concentrated at specific levels of income that they collectively can reshape the distribution of income. This is a mechanic result, for instance, when the probability of receiving transfers at a given income level from program j after ther reform (the denominator) is sufficiently small, whereas the probability of receiving program benefits was larger before the reform occurred.

3.3 Multiple factor decomposition

The logic described for the simplest case of one policy can be generalized to the multiple factor decomposition. Now, there are J policies denoted by $(\tau_t^1, \ldots, \tau_t^J)$, and X is a vector of other determinants of income. To simplify notation denote the observed density function $f_t(y_t, \tau_t, X_t)$ as $f_t(y, \tau, X)$ for any t and the counterfactual density describing the distribution that would have prevailed in 2006 had policy τ_j remained as in 1996 by $f_{06}^{C_j}(y; \tau_{96}^j)$. Then,

$$\begin{aligned} f_{06}^{C_j}(y; \tau_{96}^j) &= \int f_{06}(y|X, \tau) \cdot dF(X_{06}, \tau_{06}^{-j}, \tau_{96}^j) \\ &= \int\int f_{06}(y|X, \tau) \cdot dF(\tau_{96}^j|X_{06}, \tau_{06}^{-j}) \cdot dF(X_{06}, \tau_{06}^{-j}) \\ &= \int\int \psi(\tau_j|X) \cdot f_{06}(y|X, \tau) \cdot dF(\tau_{06}^j|X_{06}, \tau_{06}^{-j}) \cdot dF(X_{06}, \tau_{06}^{-j}) \end{aligned} \tag{7}$$

where $\psi(\tau^j|X)$ is the reweighting factor that reflects changes over time on the probability distribution of policy τ^j:

$$\psi(\tau^j|X) = \frac{dF(\tau_{96}^j|X_{06}, \tau_{06}^{-j})}{dF(\tau_{06}^j|X_{06}, \tau_{06}^{-j}).} \tag{8}$$

When decomposition factors are binary variables, $\psi(\tau^j|X)$ can be re-expressed, for transfer program j, as the propensity score function in expression (6).

3.4 Density estimation

I use kernel density methods to estimate factual and counterfactual income distributions. The kernel function is a smoothing weighted histogram that assigns weights to observations within each bin (the width of the histogram's rectangles). The bandwidth of the bins are selected based on the distance between observed points, and allowing the bins to overlap. The general kernel probability density estimator function is:

$$f_t(y) = \sum_{i=1}^{N} \frac{\omega_{i,t}}{h} K(\frac{y - Y_{i,t}}{h}) \tag{9}$$

where h denotes the bandwidth, $K(.)$ is the kernel function, $(Y_1, Y_2, ...Y_N)$ is a drawn random sample of the random variable y (income) in year t, and $(\omega_1, \omega_2, ...\omega_N)$ are the sampling weights for the N sample observations.[22]

The estimator of the counterfactual density function in expression (7) for a policy τ_j is obtained entering the reweighting factor estimate into the kernel density function in (9),

$$\hat{f}_{Y_t}^{C_j}(y) = \frac{1}{N_t} \sum_{i=1}^{N} \frac{\omega_{it} \hat{\psi}_{\mathbf{j}}(\tau_j \mid X)}{h} K\left(\frac{y - Y_{it}}{h}\right)$$

where the propensity factor reweighting function enters in the form of sample weights with ω_{it}.

The main advantage of nonparametric kernel density estimators is that it is not neccessary to impose any restrictive functional form on the underlying income generating function. Nonparametric methods let the data unveil possible offsetting effects of concurrent policy changes that might have been concealed by parametric estimation methods.

A second advantage of using nonparametric methods is the flexibility to describe distributive changes using any distributional statistic. Following Fortin et al. (2011), consider ν to be a distributional function representing a single-valued measure of the distribution $f(\cdot)$, such as the Gini coefficient. Differences of the distribution between the date groups can be measured with ν and are denoted by Δ_O^{ν}.[23] Therefore, it is possible to describe what happens to the distribution at various percentile groups or at single-valued distributional statistic such as the

[22] I use the Adaptive-kernel estimator $f(y) = \frac{1}{Nh} * \sum_{i=1}^{N} K \frac{1}{w_i}(\frac{y-Y_i}{hw_i})$, which assigns smaller bandwidth in areas with distant observed points or with low concentration of points. The optimal bandwidth is selected by an adaptive and iterative bandwidth selection process.

[23] The Oaxaca-Blinder decomposition is a special case where the distributional measure is the mean of the distribution ($\nu = \mu$).

Gini coefficient, the 90-10 percentile ratio, and other inequality statistics.

3.5 Sequential decomposition

Since programs were reformed concurrently and their effects gradually occurred, it is necessary to separate out the effects of each program taking into account the four possible sources of program variation that may be driving changes in the distribution. The overall decomposition of differences in income distribution between 2006 and 1996 into the four selected transfer programs ($J = 4$) is obtained by sequentially adding and subtracting each counterfactual distribution to the observed difference in income distributions. For the vector of programs considered here, denote $\tau = (\tau^1, \tau^2, \tau^3, \tau^4)$ =(Child Benefits, Old Age Security, Social Assistance and Employment Insurance)= (C, O, S, E).

Then the overall decomposition is expressed by:

$$\begin{aligned}
\Delta_O = & [f_{06}(y) - f_{06}^{C_x}(y; X_{96})] && X \\
& + [f_{06}^{C_x}(y; X_{96}) - f_{06}^{C_x,C}(y; X_{96}, \tau_{96}^C)] && CB \\
& + [f_{06}^{C_x,C}(y; X_{96}, \tau_{96}^C) - f_{06}^{C_x,C,O}(Y; X_{96}, \tau_{96}^C, \tau_{96}^O)] && OAS \\
& + [f_{06}^{C_x,C,O}(y; X_{96}, \tau_{96}^C, \tau_{96}^O) - f_{06}^{C_x,C,O,S}(y; X_{96}, \tau_{96}^C, \tau_{96}^O, \tau_{96}^S)] && SA \\
& + [f_{06}^{C_x,C,O,S}(y; X_{96}, \tau_{96}^C, \tau_{96}^O, \tau_{96}^S) - f_{06}^{C_x,C,O,S,E}(y; X_{96}, \tau_{96}^C, \tau_{96}^O, \tau_{96}^S, \tau_{96}^E)] && EI \\
& + [f_{06}^{C_x,C,O,S,E}(y; X_{96}, \tau_{96}^C, \tau_{96}^O, \tau_{96}^S, \tau_{96}^E) - f_{96}(y)] && Residual
\end{aligned} \tag{10}$$

where $f_{06}^{C_X,C,O}(y; X_{96}, \tau_{96}^C, \tau_{96}^O)$ is the counterfactual density that would have prevailed in 2006 had X, programs τ^C and τ^O remained as in 1996. The unexplained differences between the observed income densities in 1996 and 2006 are represented with a residual term enclosed in the last square brackets.

Decomposition estimates are sensitive to the order when there are more than two factors. By construction, the above decomposition is inherently path dependent or *non order invariant*, and the effect of a given factor would depend on sequence choice for the decomposition. Like in previous studies, I reverse the order of the sequence to verify that the estimates of the program effects are robust to the order of the sequence.

The non order invariance reflects covariance across factors different than zero. I propose a simple test to verify whether a decomposition is non order invariant which may save computational effort and propose a way to define a *more educated ordering*.

Proposition 1. ***Order Invariance Condition*****:** *Let X, τ_j and τ_k be three factors of a nonparametric reweigthing decomposition. The aggregate decomposition on these factors is order invariant if the crossed-ordering product of propensity ratios $\psi_{\tau_{j,|X}} \cdot \psi_{\tau_j|\tau_{k,X}}$ is equivalent to $\psi_{\tau_{k|X}} \cdot \psi_{\tau_k|\tau_{j,X}}$*

The proof of the Order Invariance Condition is in Appendix (B). Unless the factors selected in a decomposition of more than three factors have covariance different than zero, the results will be path dependent. One possible explanation for the statistical dependence across factors can be illustrated by the presence of program interactions. For instance, consider two alternative factor ordering: when Child Benefits is introduced before Social Assistance (*A*), and when the ordering is reversed (*B*). As described in the section of program interactions, we know the requirements about the number of kids in a family to receive CB benefits did not alter, whereas SA requirements regarding the number of kids did change with the reforms – families with young kids with SA receipt were integrated to the CB after the reform and no longer received SA benefits. On this account, the ordering of the decomposition *A* and *B* would yield different results conditional on the number of kids as one of the program requirements in both CB and SA. An investigation of the robustness of the decomposition results is a required feature of the model, unless proposition holds. I present various sensitivity checks in section (5.3).

3.6 Decomposition by factor

Socioeconomic characteristics

I account for a set of individuals characteristics X through two channels. For the first channel, a subset of characteristics X^j reflect the eligibility requirements of program τ^j. I discuss characteristics X^j and how they affect the conditional probability of receiving benefits from government program τ^j in the section devoted for each program. As proposed in DiNardo et al. (1996), the reweighting decomposition approach is straightforward to apply using binary factors, since the ratio of marginal distributions can be easily expressed into a propensity function based on observable characteristics using the Bayes' rule. The second channel directly accounts for changes in socioeconomic characteristics X. I follow Daly and Valetta (2006), by creating eight cell groups, indexed with l, resulting from interact three important determinants of income: sex, married or living under common law, and having completed college or university studies.[24]

The density for group l at date t, e.g. married men with university, is denoted by $f_t(y \mid X_l = 1)$. I use the unconditional probability for an individual of being in a cell l as a weighting factor to represent the income distribution of all socioeconomic groups at once. Then, the marginal distribution of income on X is expressed as:

$$f_t^X(y) = \sum_{l=1}^{8} \omega_{l,t} \cdot f_t^X(y \mid X_{l,t} = 1) \tag{11}$$

where $f_t^X(y)$ is the weighted sum of all cell sub-densities, and $\omega_{l,t}$ is the probability of being in group l at date t, which describes the proportion of in cell l.

The income density for each socioeconomic group and the weighted sum of the income sub-densities are plotted in figure 4. The weighted sum of all cell sub-densities is the dotted line that overlaps almost perfectly the income density of all individuals without accounting for their socioeconomic attributes. The income density of single women with high school (low education) in 1996 was concentrated below the income 20 thousand dollars, whereas in 2006 the modal income level for the same group is more spread out.

The counterfactual density of income that would have prevailed in 2006 had the distribution of socioeconomic attributes remained as in 1996, $f_{06}^{C_X}(y; X_{96})$, is defined by the sum of

[24] As robustness checks, I also examined X controlling directly for additional comprehensive set of socioeconomic characteristics including age, hours of work, experience, number of hours worked and province in a linear regression model without creating cell groups, finding smaller results than through the sex-university-married cells. It's likely that offsetting effects between characteristics changes the income distribution. I estimated the marginal densities using alternative type of cells finding no significant differences.

cell sub-densities as in (11), reweighted with the factor $\psi_X(X_l) = \frac{dF(X_{l,96})}{dF(X_{l,06})}$.

$$f_{06}^{C_X}(y; X_{96}) = \sum_{l=1}^{8} \psi(X_l) \cdot \omega_{l,t} \cdot f_{06}^{X}(y \mid X_{l,06} = 1) \tag{12}$$

The reweighting factor $\psi_X(X_l)$ is derived from the propensity score obtained from using binary socioeconomic groups, which replaces the probability of having the characteristics of a cell l of 2006, by the ones in 1996.

$$\psi(X_l) = X_l \cdot \frac{Pr(X_{l,96} = 1)}{Pr(X_{l,06} = 1)} + (1 - X_l) \cdot \frac{Pr(X_{l,96} = 0)}{Pr(X_{l,06} = 0)} \tag{13}$$

Note that the unconditional probability for an individual of being in cell l at date 1996 (or 2006), $Pr(X_{l,96} = 1)$ is equivalent to the weighted sum of type l individuals in 1996 divided by the weighted sum of type l individuals in both dates 1996 and 2006 – the full support set.

Child Benefits

The propensity score function for the Child Benefit System compares the probability of receiving benefits in 1996 and 2006 having the same observable individuals characteristics $X^{CB} \subseteq X$, including binary variables for sex, married or under common law arrangement, living with dependent children under 17 years old, province, full-time employment, education degree, sex interacted with education degree, and continuous variable for experience and quartic of experience and working hours. These variables refer to eligibility criteria for receiving CB benefits.

$$\psi(\tau^{CB} \mid X) = \tau^{CB} \cdot \frac{Pr(\tau_{96}^{CB} = 1 \mid X_{96})}{Pr(\tau_{06}^{CB} = 1 \mid X_{06})} + (1 - \tau_{CB}) \cdot \frac{Pr(\tau_{96}^{CB} = 0 \mid X_{96})}{Pr(\tau_{06}^{CB} = 0 \mid X_{06})} \tag{14}$$

where τ^{CB} is a binary variable taking the value of one if the family received Child Benefits, and zero otherwise.

To examine the redistributive effects associated to changes in the recipiency or take-up of Old Age Security program, Social Assistance and Employment Insurance respectively, I estimate the counterfactual density resulting from holding socio-demographic characteristics, and the respective program as in 1996. The mechanics are the same as for the Child Benefit program, leading to the OAS reweighting propensity function with $X^{OAS} \subseteq X$, including binary variables for sex, married or under common law arrangement, education degree, province, sex interacted with education degree and continuous variables for age and age squared, experience and quartic of experience. Like with previous programs, I estimate the reweighting factor to adjust the 2006 income density accounting for changes in SA and EI receipts. Finally, the difference between the observed density in 1996 and the counterfactual of the last factor is the residual of the decomposition. The findings from the counterfactual exercises are interesting and they are discussed in section 5.

4 Data description

For this analysis, I use the confidential files of the Survey of Labour and Income Dynamics (SLID) collected by Statistics Canada. The SLID is series of six-year overlapping panels, surveying approximatedly 15,000 households per year from 10 Canadian provinces excluding Indian reserves. A crucial feature of the SLID is that, besides standard demographic and labour information, tax and transfers data are reported directly from tax files for over 80% of respondents who granted consent to Statistics Canada to access directly from filed T1 tax form and supplementary administrative files. The remaining 20% answers tax questions from the SLID questionnaire. This feature enables me to identify actual receipt of program benefits rather than simulate recipiency based on imputed eligibility criteria, under the assumption that all eligible individuals actually receive benefits (Frenette et al., 2009).[25]

The observation unit is a family represented by the main income earner and spouse or common-law partner. Socioeconomic characteristics like age and sex are those of the main income earner, but information on income, taxes and transfers refer to the the sum of all family members expressed in family equivalent units. Families with negative income or above the 99.5 percentile were removed for consistency of the estimates and to preserve the confidentiality of the SLID survey. Full-time students are excluded. The final sample (non-student

[25]This is considered to be a rather unrealistic assumption Currie (2004)

main income earners aged 18-65 and spouse living in the 10 Canadian provinces) consists of 37,000 and 35,000 observations in 1996 and 2006 respectively.

Unless specified otherwise and for what remains in the paper, *income* refers to the real after-transfer family income expressed in adult equivalent units and in 2010 dollars. The family equivalent scale allows me to compare income across families with different size and composition.[26] The after-transfer income is the sum of market income (earnings and other market income such as investment income, pension, alimony and other taxable income before taxes and transfers) and all government transfers such as the CB, OAS and Guaranteed Income Supplement / Spousal Allowance, SA and Provincial Income Supplements, EI, Worker's Compensation Benefits, Canada/Quebec Pension Plan Benefits, and the GST-Credit. I analyze after-transfer income rather than disposable income (after tax *and* transfers) because it partly isolates the effects of taxes – except spillover effects of tax changes on the take-up of benefits and on income – and allows me to focus on the effect of social programs on the income distribution. As a robustness check, I also estimate the model using disposable income which yields similar results.

4.1 Summary statistics

Annual average of income, employment and socio-demographic characteristics of couples in the sample are reported in table 3 for the 1993-2009 period.

Over the period of study, the number of children living in Canadian households is diminishing. Reflecting the expansive immigration policies, the proportion of families with at least one member identified as minority group has expanded. There is an increase in education of the main income earner and in the fraction of households living in main cities (Census Metropolitan Areas), while union status reflect the secular trend in de-unionization and hours of work that follow the business cycle.

[26]To account for economies of scale within a family (an additional family member increases expenses but at a lower rate), family income is scaled assigning weight of 1 to the oldest member of the family, 0.4 to other adults older than 16 years, and 0.3 to each person younger than 16. This scaling technique was formerly used by Statistics Canada, however an equivalente but simpler technique based on the square root of the family size is currently employed.

Employment is the main income source for 70% of Canadian households. Between 18% and 14% of households rely on governemnt transfers as the main source of income and, consistent with demographic trends, an increasing fraction of families claim retirement pensions as their main source of income. Both, the average real market income (pre-fisc) and real disposable income (post-fisc) is rising. Income tax also increased but government transfers have remained the same.

The SLID has some limitations regarding the income representativeness of the sample. Families in the two tails of the income distribution are underrepresented which may underestimate any inequality measure of the distribution. Frenette et al. (2007), for instance, argues that the census is the most reliable data source to measure income inequality. However, I believe that the advantages of having accurate tax information, overcome these disadvantages.

5 Results

Decomposition results of a benchmark ordering sequence are discussed, and after compared to estimates decomposed in reverse order.

Results illustrate the importance of separating the effects of multiple policy reforms that occurred simultaneously within a period. Accounting for the effects of all transfers on changes in the distribution may conceal the possible opposite effects of different programs. Furthermore, using parametric methods may obscure the effects of programs that affect the distribution only locally. In contrast, allowing the data to speak for themselves, nonparametric methods offer a more precise description of the effects without any assumption about the functional relation between between income and factors producing estimates with *distribution-free* properties of the estimates. All these features of the model are appealing because, (i) they point out what new hypothesis can be formulated and further examined, and (ii) they provide a practical analytical tool to assess the distributive effectiveness of policies.

5.1 Benchmark decomposition sequence

The decomposition results of the log income density difference between 1996 and 2006 from expression (10) are graphically described in figure 5. It shows (*i*) what happens and where in the distribution, and (*ii*) the locus in the distribution where programs have opposite effects.

The distributive effects of program recipiency are identified by the difference between the counterfactual densities for each program, which are plotted in figure 5. The vertical axis refers to density differences and the horizontal zero line indicates that there were no changes in the distribution between the two years. Lines above zero show increases in the concentration of families in 2006 relative to 1996. The solid line in all panels of figure 5 is the total difference to be explained: 2006-1996. The dashed lines are differences between two counterfactual densities stated in each line of equation 10. These differences can be interpreted as the marginal effect of a given explanatory factor, which varies over the entire distribution. Regions where the dashed line deviates from the solid line in figure 5 describe the portion out of the total differences in the densities explained by the considered factor. Regions where the dashed line deviates the most are the income levels of families most affected by a factor.

Figure (5a) show that changes in the socio-demographic of households explain a modest part of total difference in the distribution: an increase in the concentration of families at the bottom of the distribution up to 11 units of log of income (or 60,000 dollars = e^{11}), and a reduction in the concentration of families above this income level. Hence, socio-demographic changes have worsened the income distribution.

The marginal effect of changes in CB recipiency is represented in figure (5b). There is a significant decline in the concentration of families with income in the range of 9.5 to less than 11 units of log of income ($13,000 to $50,000). Similarly, there is an increase in the mass of families with income above this level and up to 12 log of income ($160,000). The rise in the generosity of child benefits and legislation changes to increase recipiency of benefits by low-income families may explain these effects. CB reaches a relatively large portion of families with different income levels, and its effects are found along the entire distribution.[27]

[27]The income threshold of CB benefits depends on the total number of children eligible which can amount up to 70,000 dollars of 2012 for a family with 8 eligible kids. http://www.cra-arc.gc.ca/bnfts/cctb/cctb11byclc-eng.html

Marginal effects of OAS are plotted in (5c). As expected very modest changes occurred since the legislation to OAS did not change during the period of study. The small difference in the density of families with OAS receipt can be explained by age through the way seniors have their transfers assigned, as explained in section 2, and the aging of the population. Old Age Security explains modest increases of the fraction of families with income between 10 and 10.6 log of income ($22,000 to $40,000) and a not obvious decline of families with income around 11.2 log of income (73,000 dollars).

The effects of Old Age Security should be interpreted with caution. OAS per se is not causal of the decline of high-income families and the increase of those with low-income, it is rather associated to the rise in the proportion of retirees (the aging of the population) whose income inherently deteriorates with retirement. In the absence of OAS, the rise of low-income families and the decline of high-income ones would have been greater, since the provision of income security attenuates the effects of retirement.

The distributive marginal effects of Social Assistance are displayed in figure (5d). These indicate that changes in SA recipiency resulted in two modal income levels. The spike centered around 10.2 of log of income ($27,000) reflects the fact that in 2006 families with this income were much less likely to receive SA than in 1996. These dramatic results could have been also driven by the cuts to SA benefits.[28] Moreover, the overall reduction in SA receipt is associated somehow with an increase in income in the region between 10.2 and 11.3 log of income ($ 27,000 to $80,000). It is possible that a portion of such increase is due to the incentives to work that the SA promoted among the employable individuals receiving benefits before the reform. The spikes reflect factor effects locally concentrated around an income level.[29]

The distributive marginal effects of changes in Employment Insurance recipiency are

[28] As an example of the extent of the cuts considere the province of Ontario, which in had the most generous SA. These included (in addition to other assistance) a shelter allowance to support rental or mortgage payments up to 450 dollars per month and the possibility that two unmarried adults living in the same household could claim SA benefits independently. Both benefits were greatly reduced after the reforms. In-depth analysis regarding the effects of SA changes is out of the scope of this study. See Kneebone and White (2009) for detailed description of changes to SA.

[29] A good example of factor effects centered are be found in DiNardo et al. (1996) original work, where the effects of the decline in the real minimum wage are locally concentrated around the minimum wage level.

plotted in figure (5d). Changes in EI recipiency importantly reduced the concentration of low-income families (up to 9.9 log of income or \$20,000) and increased that of families with income in the range of 10.4 and 11.2 log of income (\$33,000 to \$73,000). Reforms to EI appear to counterbalance the effects of SA, which can be explained by the integration and interaction of these two programs. The SA administered various *Separate Assistance-Linked* benefits before the reform. One of them considered additional income assistance for workers with modest employment earnings and with dependent children as incentive to continue in the work force. After the SA cut-offs, it is possible EI covered eligible workers previously assisted by SA.[30]

The portion of the total density differences that remains unexplained after adjusting by changes in sociodemographics and in the four transfer programs, the residual, is shown in figure (5e). Residual or unexplained elements reduced the fraction of low income families (from 9.1 to 9.9 log of income or from \$9,000 to \$20,000) and increased the rise of families with income between 10.8 and 11.3 log of income (\$50,000 and \$80,000). Changes to the personal income tax in 2001 are good candidate to explain the residual. By studying differences in densities of after-transfer income, I abstract from great part of the effect of tax changes. However, spillovers derived from tax changes are not examined. Nevertheless, we check whether the effect of transfer programs are robust to changes in the definition of income. These results are presented in the next section.

An important feature of the decomposition method used is the non order invariance of the decomposition results. It is possible that decomposition effects are influenced by the ordering if the factors are not independent. This is particularly the case when a factor early in the sequence has relatively large propensity score function values around a region of the distribution (local large effects), making more likely for subsequent factors to display also extreme effects. To check the robustness of the results to this possibility, I reverse the order of the decomposition as is commonly done in previous applications of the method (Fortin et al., 2011) and find that the effects of programs are robust and report only minimal variation,

[30]Further examination of the channels by which EI affected the income distribution merit attention outside the scope this study.

which I discuss in the next section.

5.2 Distributive measurements by factor

Inequality measures

Program effects are summarized with various measures of distributional statistics. Table 4 describes income disparities by reporting different measures of inequality, each one revealing where in the distribution the gaps between families are widening up. A supplementary table (5) shows the average income of families at a varying percentile of the distribution.

The first two rows of table 4 outline the inequality of income in 1996 and 2006. The first column shows the 90-10 percentile ratio, indicating that in 1996 the average income among the top 10 percentile was 4.8 times greater than that at the bottom 10th. In 2006, this same ratio increased to 5.02, widening the income gap between the two tails by 0.16 points (3.3%). The second column compares the income between the top 10 percentile and the median income families, which ratifies an increase in inequality from 1.9 times more income earned by the top 10 relative to the 50 percentile in 1996, to 2.1 in 2006; an increase of 6.2% (0.12 difference points). The third column shows declining income disparities among families in the first half of the income distribution; as the ratio between the median and the bottom 10th percentile fell 2.7% points, from 2.5 to 2.4. These three columns provide evidence of the rising concentration of income at the top half of the distribution.

Inequality measures in columns (4) to (6) vary in the region of the distribution where they are more sensitive to income differences. The Gini coefficient in column (4) is more sensitive around the modal level of income, and grew 3%, from 0.33 to 0.34. The mean log deviation is more sensitive to differences at the bottom of the distribution, and increased 4% relative to 1996. The Theil's index - show the highest sensitivity to changes around the average income - grew 6% as shown in column (6). The Atkinson index with parameter value of 0.5 reported in column (7), the most responsive measure to differences at high income levels, rose 6% (from 0.096 to 0.091). These supplementary inequality measures are consistent with the increasing concentration of income at the high end of the distribution, and less so at the bottom.

Transfer programs are expected to reduce inequality proportionally more in the lower half of the distribution and much less among those above the median income. Program effects are shown in the second section of table 4. They indicate the difference in each inequality statistic calculated over the two (counterfactual) distributions considered in each line in equation (10). More interestingly, the second row in parenthesis, shows the normalized share of the total absolute difference between 1996 and 2006 explained by a given factor. The sum of the absolute value of all factor shares plus the residual within a column equals 100. The sign indicates the direction of the change and the value represents the relative contribution of a given program to the total change. A comparison of results by program (results in the same row) refers to where in the income distribution the factor effects are more pronounced.

Consider the decomposition results described by the Gini coefficient in column (4). They indicate that changes in the composition of families based on the sex-education-marriage status of the family head increased inequality, but contributed only 4.4% in explaining overall increases in the Gini coefficient. Changes in child benefit and OAS recipiency actually reduced inequality, although the later was negligible. Changes in SA recipiency, on the other hand, had an important effect in contributing towards increasing inequality, being responsible for 25% of total absolute changes in the Gini coefficient. These were counteracted by changes in EI, which account for 37% of total absolute changes in inequality. Other unexplained changing factors (unaccounted for) also contributed to reduce inequality as measured by the Gini.

The relative contribution of each factor in explaining overall changes in the distribution described for the Gini coefficient are similar if we use other measures of inequality with a few exceptions. For instance, in explaining the reduction in the gap of the median to the bottom 10 percent of the distribution, CB plays a more important role, but EI only a minor one. Both, however, are dwarfed by the relative importance of SA and residual factors, which greatly *increased* inequality in this regard. Another discrepancy can be observed when looking at the inequality between the top 10% of the income distribution and the median. SA is a major factor explaining the widening of this gap, whereas the importance of the residual factor becomes negligible.

Reversing the sequence of factors

The non order invariance is a downside of decomposition methods which may be pronounced if the sequence consists of more than three non-independent factors. To check whether the ordering of decomposition influences factor effects, the convention in the literature is to repeat the analysis in reverse order, which I compute starting with EI, followed by SA, then OAS and CB at last. (DiNardo et al., 1996 and Fortin et al., 2011). Results from reversing the order of programs in the decomposition are presented in panel B at the bottom of table 4. The locus and relative contribution of programs are robust to the reversed sequence of the decomposition, however the magnitude of the effects is different.

In general, decomposition results are found to be qualitatively robust to the factor sequence. Focusing again on the Gini coefficient in column (4) of table 4, panel B, we see that the effect of EI is in the same direction, albeit smaller, than the effects observed with the benchmark decomposition. SA has a relatively larger effect in the reverse decomposition as does OAS. Child benefits have a negligible effect and the residual has a smaller contribution as well.

Locus of program's greatest effects

Results in table 5 show the average income of families at various percentiles expressed in thousands of dollars. The first two rows show average income in 1996 and 2006 respectively within percentile in the distribution. In 1996 average income among the 5% poorest families was 11.5 thousand dollars, and 91.7 thousand dollars for the top 95 percentile. Income at the two tails showed the largest growth (22%) in 2006. Median income grew the least through the period (13%) to reach 43 thousand dollars in 2006.

To identify in which region of the distribution programs had their greatest effects, panel A in table 5 reports the average income of each percentile according to the counterfactual density function. The (normalized) percentage change implied by a given program out of the total absolute change for the percentile is shown underneath in parenthesis.

Consider for instance the effect of program changes on the bottom 10% of the distribution. The effect of socio-demographic characteristics resulted in an increased income of 1.3 thousand dollars for the lower end of the distribution, which accounted for around 4.5% of the total absolute change for the percentile. Changes in sociodemographic characteristics had a uniform effect on the distribution. Changes in the recipiency of CB significantly incresed income at all levels of the distribution. This suggests that changes in CB induced increased take-up of benefits for a broad espectrum of income levels. Changes in CB recipiency accounted for 14% to 33% of all changes at each percentile. The next row shows the effect of changes in the recipiency of OAS, which was as expected, very small since it did not involve an actual change in the program. It did, however, contribute to increasing income at the bottom of the distribution. Note that this is not a result of the program itself but mainly of the aging of the population. Next row shows the effects of changes in the recipiency of SA, which reduced income at all levels of the distribution and is the factor explaining most of the changes at percentile 25 and 50. The larger effects of these changes can again be observed at the median and for the top of the distribution. EI (next row), on the other hand, has the opposite effect, increasing income at all levels of income, but particularly at the bottom. Finally, the last row of the panel shows the effects attributed to the residual, which contributed to reduce income at all levels of the distribution, particularly at the bottom.

Focusing on which factor had the largest effect for a given percentile, EI had the largest effect at the median and bottom of the distribution, whereas changes in Social Assistance have the largest effects at the median of the distribution. OAS explains little of the overall change in income for all percentiles. The effect of changes in unexplained factors are concentrated at the bottom with income reduction and a relatively minor increase of income at the top end of the distribution.

The bottom panel of table 5 reports results from reversing the sequence of factors. As for the results presented in terms of inequality measures, results are qualitatively robust to the ordering of the decomposition. Employment Insurance increases income proportionally more at the first half of the distribution, explaining for instance 22% increase in income out of the total absolute change in income at the 10th percentile. Social Assistance reduces income disproportionally more at the low end of the distribution than among families at the top, which hinders the reduction of income disparities. An important difference with the benchmark ordering is the change in the direction of OAS effects, as it appears to increase income at any income level particularly below the median income. Recall however, the drawback of decomposition methods when the sequence has more than three non independent factors, and is suggested to interpret results of the last factors in the sequence in relative terms (the numbers in parenthesis). If the results are equal unconditional of the ordering, like SA, then the analyst could interpret the differences in income in absolute terms – the differences in income in the first row of each factor.

5.3 Extensions and robustness checks

In order to verify that the program effects are robust to the definition of income, in this section I decompose differences in the density of disposable (after-tax and transfers) income. In 2001, changes to the personal income tax brackets occurred: a new additional income bracket at high income level was included and the three existing tax brackets were relocated at different income levels.[31] The trends of income inequality using after-transfer income or disposable income in figure 1 precisely cross in 2001, suggesting the tax reform itself influenced the income distribution. Appropriate microeconomic modeling of the tax reform merits meticulous analysis and its inclusion would surpass the dimensionality of decomposition methods. I here acknowledge the possible distributive effects of changes to the tax system to be examined in future work; and for now, I check whether decomposing differences in disposable income alter the results of program effects derived from decomposing after-transfer income.

Results in table 6 show that in general, transfer programs effects are robust to either disposable income or after-transfer (and before taxes) income. Results preserved the relative contribution by program in explaining differences in the income distribution defined as in the

[31]To simplify the tax reform: there were three tax brackets in 2000, tax rate of 17% with income lower than 30,000, 25% for the amount exceeding 30,000 and up to 60,000 dollars, and 20% for more than 60,000. After the reform, the federal tax rates for 2001 dropped at lower income levels, and an additional bracket was added between 60 and 100 thousand dollars: 16% in the first bracket up to 30,754 dollars of taxable income, 22% on the next 30,755 dollars, 26% between 61,509 and 100,000 and 29% over 100,000 dollars. Frenette et al. (2009) discuss the role played by the reduction of federal surtaxes reduction in 1990 and of provincial tax rates in the 1995-2000 period in changing the income distribution.

benchmark case using after-transfer. There are however a few differences I describe in this section.

As expected, the tax system significantly redistribute income. The first row of tables 5 and 6 report income levels in 1996. Consider column (3) in the two tables. Families at the 25 percentile in 1996 have \$24,020 of after-tax income, and 21,350 of disposable income. Similarly, families at the top 90 have \$74,130 after-transfer income and \$56,900 disposable income. There is evidence that personal income tax is progressive as the decline in average income that results from comparing the two tables is proportionately larger the greater the percentile. Thus, families at percentile 5 have a net benefit of taxes of \$20 (11.49-11.51), at the 10th percentile families paid \$80 on taxes, median income families observe \$6,450 reduction (37.80-31.35), and top 95 paid on average families average \$23,540 dollars. Unconditional to the income definition used, the evidence in the two tables show that income gaps grew from 1996 to 2006 and that such disparities are more spread out the higher the percentile. Overall, redistribution does reduce income disparities; however, the question to ask is how effective the tax system has become in doing so over time.

The distributive effects of transfer programs identified by changes in the number of families receiving benefits are qualitatively comparable. In strict sense, comparing 5 and 6 may not be sensical as they refer to different type of distributions, yet, the relative contribution of transfer programs in explaining over time distribution differences should not change dramatically when using other income definition such as disposable income. I find changes in sociodemographics (sex, married, and college or university education) are concentrated at the top 90 an 95 percentiles (comparison by row) in the two tables, 4.5 of after-transfers income and 7.2 of disposable income). Child Benefits' largest effects increase income of families at the 10 and 25 percentiles as shown in the two tables. The income decline associated to changes in the receipt of Social Assistance are concentrated at the low end of the distribution, whereas Employment Insurance seems to *compensate* low-income families with an income increase that amount nearly the loss associated to SA changes. Being EI the last factor in the sequence preceded by the large and locally concentrated effects of SA, likely explains its magnified effects. Income differences explained by EI in this table should be interpreted with caution; yet, after checking results from the reverse decomposition, EI effects are mostly clustered around the median and first half of the distribution.

The relative contribution of programs at given percentile levels can be derived from comparing the percentage change explained (in parenthesis) by factor out of the total difference in income densities from 1996 to 2006. The two tables show SA and EI explain most of the income change and preserve the same signs. At percentile 25 and 50, the magnitude of CB and SA are close but they act in opposite direction, CB increasing income and SA reducing it. Note the share explained by unaccounted factors is larger in the decomposition of disposable income than in the after-transfers income. This makes sense as the dependent variable in the first case is income after tax were paid, so the effect of transfers accounts for only part of the differences in disposable income as income tax is not a decomposition factor in my study. Overall, the distributive effects of programs are robust to the change in the income definition. This suggests that changes in the distribution of other unaccounted factors are not sufficiently influential to alter the effects that programs have in the income distribution.

6 Conclusions

This paper attempts to improve our understanding of the redistributive effects of transfer programs. Historically, separating the redistributive effects of concurrent policy reforms is challenging. Most studies on the distributive effects of fiscal policies provide mechanical measures of simulated changes to fiscal policies. Nonetheless, these effects are assumed to be exogenous to behavioral response and are based on simulations that assume full tax compliance, and 100% program take-up rates – target population is fully reached by the program.

I propose the application of a nonparametric decomposition method developed by DiNardo et al. (1996), to assess the effect of changes to transfer programs that influence families' income. The distinctive piece of the model is the creation of density counterfactuals that enables me to identify the distributive effects of changes to the receipt of program benefits. In order to show the application of this method, I decided to analyze the Canadian case because of the availability of a rich database, and because major transfer programs were concurrently reformed. Using the Survey of Labour and Income Dynamics (SLID), I examine the probabilities of receiving benefits by a Canadian family before and after the program reforms. Families perceive the program changes and their responses are summarized by a propensity score function, called the reweighting factor, which intuitively reflects the change in the likelihood of receiving program benefits due to the program reforms, once other observed family characteristics are kept constant over time.

The appealing features of this model rely on its descriptive capacity: the results describe what the program effects are (increasing or decreasing income) and where in the distribution these effects are concentrated. It also separates out the relative contribution of each factor from reforms to multiple programs occurring simultaneously in the same period. This model may uncover policy effects not yet identified by conventional parametric methods, particularly when a program affects the distribution locally (when family responses are sufficiently concentrated at specific income levels), or when programs have opposite distributive effects. Despite the fact that the model does not recover behavioral relationships or structural parameters between income and the transfer program, the results from this approach points out what hypothesis are there to be explored, for instance, whether reforms to the Social Assistance program lead to targeting improvement, whether the take up of SA benefits have increased, or whether Child Benefits is efficient while reaching high-income families.

The lack of discernible downsizing in income inequality during a period when a series of reforms to transfer programs were implemented in Canada, suggests that perhaps, reforms to the tax and transfer system may have influenced the determination of income in a way that makes the tax system less effective in reducing income disparities. I find results consistent with the literature regarding the declining equalizing effectiveness of the tax and transfer system, but unlike previous studies, I describe in detail where in the distribution each program had its greatest effects, and what is the relative contribution of the each program in explaining the differences in the income distribution from 1996 to 2006. I find Child Benefits reduces the concentration of low-income families but increases income disparities between the percentiles. Changes in the receipt of Old Age Security benefits, although are not preceded by legislative changes, show modest distributive effect reducing concentration of low-income families and rising the cluster of medium income families. Employment Insurance reduces inequality mostly at the low half of the distribution, whereas changes in the receipt of Social Assistance dramatically increase income disparities by rising the share of low-income families relative to 1996.

Although the evidence is based on Canadian data, the analytical technique for estimating the redistributive effects of transfer programs may be employed for other economies. Perhaps some of the next steps in understanding the evolution of income inequality would characterize a menu of policy instruments capable to amend the increasing trend of after-transfer income inequality under the lens of economic efficiency.

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A Appendix: Tables and Figures

Table 1. Summary of Program Reforms

Program	Spending ($billions)	Annual Benefit ($thousands)	Changes	Recipiency change
Child Benefits	10	3[a]	+ National Child Benefit Prg & Supplement New supplement for low-income families.	-5.6%
Social Assistance	11	12-16[b]	- Federal cuts, lower benefits, more restrictive. Transfers to provinces are combined with Health and social transfers. Provinces condition benefits to job searching, training or work	-50%
Employment Insurance	12	413 weekly	± Fewer weeks paid, requirement of previous weeks of work increased decreasing coverage for recurrent claimants.	-21%
Old Age Security + GIS	30	13.2[c]	Non legislative changes. Aging population.	+11%

Notes: Spending in current billion dollars of 2006. Annual benefit refers to the eligibility unit defined in the program.
(a) Benefits for the first child. Clawback if family income exceeds 25 thousand.
(b) SA benefits for single parent with one child.
(c) OAS benefits for single senior. Clawback if income exceeds 48 thousand, repaying 15 cents per dollar above this threshold.
Source: Data on spending for all programs is from CANSIM table 385-0009, and from the *2011 CPP and OAS Statistics Book* at http://www.servicecanada.gc.ca/eng/isp/statistics/cppstatbook/statbook2011. Author's estimates of the change in program receipt (1996-2006) based on the SLID and shown in table 2.

Table 2. Receipt of Benefits from one or more Programs: 1996-2006.

Year	Num obs	Percentage of families receiving program benefits.				Program Interactions (% of families receiving benefits from multiple programs)									
		Children Benefits (CB)	Old Age Security or Guaranteed Income Supplement (OAS/GIS)	Social Assistance (SA)	Employ-ment Insurance (EI)	CB & EI	CB & SA	SA & EI	SA & OAS	EI & OAS	CB & OAS	SA & CB & EI	SA & CB & OAS	SA & EI & OAS	CB & EI & OAS
Couples															
Difference: 2006-1996		**-5.6**	**12.3**	**-47.1**	**-20.9**	**-16.6**	**-53.9**	**-58.1**	**-38.2**	**-2.4**	**42.5**	**-65.2**	**-12.5**	**-58.1**	**18.8**
1996	37,077	36.4	18.0	10.2	24.5	12.5	4.6	2.4	2.5	1.3	0.9	1.4	0.3	0.3	0.3
1997	36,490	35.3	18.4	9.6	22.4	11.2	4.2	2.1	2.8	1.2	1.0	1.1	0.4	0.3	0.3
1998	36,428	31.7	18.7	9.2	21.3	9.6	3.6	1.8	2.9	1.2	1.0	0.9	0.4	0.3	0.2
1999	37,427	30.9	18.6	6.9	18.9	8.5	3.1	1.4	1.6	0.9	1.0	0.7	0.2	0.2	0.2
2000	36,670	31.6	18.7	6.6	18.4	8.5	2.9	1.3	1.6	1.0	0.9	0.8	0.2	0.2	0.2
2001	36,635	31.4	19.3	6.6	19.9	9.2	3.0	1.4	1.8	1.2	1.1	0.8	0.4	0.2	0.3
2002	36,287	31.2	19.2	5.9	21.3	10.3	2.4	1.3	1.5	1.2	1.1	0.7	0.3	0.1	0.3
2003	36,372	30.0	19.2	5.9	21.0	9.7	2.3	1.2	1.5	1.3	1.1	0.6	0.3	0.1	0.3
2004	34,838	31.9	19.6	5.8	20.9	9.9	2.2	1.1	1.7	1.4	1.1	0.6	0.3	0.2	0.4
2005	34,612	33.2	19.7	5.4	19.5	9.9	2.1	1.2	1.5	1.4	1.3	0.7	0.2	0.2	0.4
2006	34,860	34.3	20.2	5.4	19.4	10.4	2.1	1.0	1.6	1.2	1.2	0.5	0.3	0.1	0.4
Std dev*100 (range in period)		(.462 - .482)	(.377 - .412)	(.226 - .302)	(.391 - .447)	(.278 - .354)	(.141 - .209)	(.099 - .161)	(.12 - .167)	(.106 - .127)	(.086 - .118)	(.068 - .122)	(.033 - .060)	(.037 - .060)	(.045 - .065)

Notes: Families receiving benefits are those where either the main income earner or the spouse received benefits. The percentage is out of all couples. The difference (2006-1996) reported the first row is the percentage change with respect to 1996. Program interactions are expressed as percentage out of all couples in the sample. Standard deviations per year are within the range enclosed in parenthesis in the last row . Author's estimates using the sample of main income earners and spouse with family income above zero based on the SLID,1996-2006.

Table 3. Summary Statistics of Canadian couples: main income earner and spouse: 1996-2006.

			Socioeconomic characteristics								Employment		Main source of income			Income (Family equivalent scaled in 2010 dollars)				
Year	N	Family equivalence scale (FES)	Male	Married	Living with kids aged <17 years	Minority	CMA	Full-time job	Union	Years of schooling, full-time equivalent	Experience	Worked hours, all jobs all family members	Wages/ salaries	Gov transfers	Retirement pensions	Market income	Gov transfers	Before-tax income	Income tax	After-tax income
Couples																				
1996	37,077	1.65	0.52	0.71	0.31	0.14	0.78	0.85	0.37	12.96	26.8	2,673	0.71	0.17	0.03	38,647	4,958	43,605	8,719	34,886
1997	36,490	1.65	0.52	0.71	0.31	0.13	0.78	0.85	0.36	13.01	27.2	2,713	0.71	0.17	0.04	39,635	4,871	44,506	8,927	35,579
1998	36,428	1.64	0.52	0.70	0.31	0.13	0.78	0.86	0.35	13.11	27.4	2,729	0.72	0.16	0.04	41,267	4,814	46,081	9,416	36,665
1999	37,427	1.65	0.52	0.71	0.31	0.15	0.79	0.87	0.36	13.18	27.5	2,727	0.71	0.16	0.04	42,014	4,602	46,615	9,248	37,367
2000	36,670	1.64	0.52	0.70	0.31	0.17	0.79	0.87	0.36	13.23	27.8	2,763	0.72	0.15	0.04	43,233	4,573	47,806	9,533	38,273
2001	36,635	1.64	0.52	0.70	0.31	0.18	0.79	0.87	0.36	13.31	28.2	2,833	0.72	0.15	0.05	44,191	4,861	49,051	8,978	40,074
2002	36,287	1.64	0.52	0.71	0.31	0.20	0.80	0.87	0.36	13.37	28.1	2,741	0.72	0.15	0.05	44,476	4,815	49,291	8,854	40,437
2003	36,372	1.65	0.52	0.70	0.30	0.19	0.80	0.87	0.36	13.43	28.3	2,791	0.72	0.15	0.05	44,339	4,810	49,149	8,916	40,233
2004	34,838	1.64	0.52	0.69	0.30	0.19	0.79	0.87	0.35	13.53	28.4	2,797	0.73	0.14	0.05	45,232	4,840	50,072	9,028	41,044
2005	34,612	1.63	0.52	0.70	0.29	0.18	0.81	0.87	0.35	13.39	28.2	2,756	0.73	0.14	0.05	45,797	4,799	50,596	9,002	41,594
2006	34,860	1.63	0.52	0.70	0.29	0.17	0.81	0.87	0.35	13.45	28.6	2,744	0.72	0.14	0.05	46,588	5,066	51,654	9,115	42,538
Std dev (range in period)		(.476 - .493)	.499 - .50	(.444 - .461)	(.444 - .463)	(.298 - .396	(.387 - .795)	(.331 - .362)	(.475 - .489	(3.43 - 3.86)	(16.8 - 17.8)	(1849 - 1955)	(.445 - .456)	(.349 - .382)	(.172 - .239)	(31191 - 5920	(6227 - 6628)	(29088 - 4845	(10456 - 1865	(2995 - 40065)

Notes: Author's estimates using the sample of couples with family income above zero based on the SLID,1996-2006. All variables are annual averages. Socioeconomic characteristics (except schooling) and the main source of income are binary variables. Worked hours is the sum of annual working hours by all family members in all jobs. Income in real dollars of 2010 and family equivalent units. Standard deviation in each year is in the range enclosed in parenthesis in the last row.

Table 4. Decomposition of Differences in After-transfer Income Densities (1996-2006), reported by Dispersion Measures.

	(1) 90-10	(2) 90-50	(3) 50-10	(4) Gini	(5) Mean log deviation	(6) Theil's index	(7) Atkinson index, 0.5
1996	4.860	1.961	2.478	0.331	0.199	0.189	0.091
2006	5.021	2.083	2.410	0.341	0.207	0.200	0.096
Total change: 2006-1996	0.161	0.122	-0.068	0.010	0.008	0.011	0.005
A. Benchmark order							
Sociodemographics$^{(a)}$	0.16	0.10	-0.04	0.01	0.01	0.01	0.01
	(3.1)	(4.6)	(3.7)	(4.4)	(4.1)	(4.7)	(5.1)
Child Benefits	-0.70	-0.10	-0.20	0.00	0.00	0.00	0.00
	-(12.6)	-(6.4)	(17.4)	-(9.3)	-(8.3)	-(8.1)	-(8.1)
Old Age Security	-0.17	-0.03	-0.06	0.00	-0.01	0.00	0.00
	-(3.1)	-(1.3)	(5.4)	-(1.1)	-(2.3)	-(1.4)	-(2.0)
Social Assistance	1.40	0.90	-0.22	0.00	0.10	0.10	0.00
	(26.6)	(44.1)	(22.2)	(25.3)	(22.9)	(24.6)	(24.2)
Employment Insurance	-1.70	-0.80	-0.05	-0.10	-0.10	-0.10	0.00
	-(32.8)	-(39.4)	(4.6)	-(36.8)	-(37.6)	-(37.9)	-(37.4)
Unexplained	1.20	0.10	0.50	0.00	0.10	0.00	0.00
	(21.8)	(4.2)	-(46.6)	(23.1)	(24.8)	(23.2)	(23.2)
B. Reverse order							
Sociodemographics	0.16	0.10	-0.04	0.01	0.01	0.01	0.01
	(3.6)	(7.5)	(2.9)	(5.3)	(4.9)	(6.5)	(6.5)
Employment Insurance	-0.89	-0.14	-0.29	-0.02	-0.03	-0.02	-0.01
	-(19.7)	-(10.8)	(22.9)	-(15.1)	-(15.4)	-(15.7)	-(15.6)
Social Assistance	1.34	0.32	0.29	0.05	0.06	0.05	0.03
	(29.8)	(25.4)	-(22.9)	(32.2)	(34.1)	(32.7)	(32.5)
Old Age Security	-1.29	-0.30	-0.29	-0.05	-0.06	-0.05	-0.02
	-(28.6)	-(23.5)	(22.9)	-(30.3)	-(32.4)	-(30.7)	-(31.2)
Child Benefits	0.25	-0.14	0.30	0.00	0.01	0.00	0.00
	(5.6)	-(10.9)	-(24.4)	-(1.3)	(3.3)	(1.3)	(2.6)
Unexplained	0.58	0.28	-0.05	0.02	0.02	0.02	0.01
	(12.8)	(21.8)	(3.9)	(15.8)	(9.9)	(13.1)	(11.7)

Notes: The percentage explained by program out of the total change in the density of after-transfer (and before-tax) income is shown in parenthesis. Generalized Entropy Inequality indexes vary in the regions of the distribution where they are more sensitive to income differences. The mean log deviation (column 5) is more sensitive at the bottom, and the Theil's Index (column 6) is more responsive around the median. The Gini coefficient (column 4) weights more families around the modal income level. The Atkinson index is most responsive at the bottom the greater its parameter is; column 5 computed with $\alpha = 0.5$ is more sensitive at high income levels. Standard errors are available upon request; no reported because of space limitations. Author's calculation using the SLID 1996, 2006.

Table 5. Decomposing differences in After-transfers Income Densities (1996-2006), reported by percentile.

	(1) 5	(2) 10	(3) 25	(4) 50	(5) Mean	(6) 75	(7) 90	(8) 95
1996	11.5	15.25	24.02	37.8	43.31	55.34	74.13	91.66
2006	14	17.76	27.78	42.81	50.96	64.11	89.16	111.71
Total change	2.5	2.51	3.76	5.01	7.65	8.77	15.03	20.05
A. Benchmark order								
Sociodemographics	1.30	1.30	1.30	2.50	4.40	5.00	8.80	11.30
	(4.5)	(4.5)	(4.5)	(4.5)	(4.5)	(4.5)	(4.5)	(4.5)
Child Benefits	3.80	6.30	10.00	11.30	11.70	12.50	16.30	21.30
	(13.7)	(20.8)	(27.6)	(20.4)	(23.4)	(21.3)	(32.5)	(25.8)
Old Age Security	1.25	0.00	-1.26	-1.25	-1.83	-1.25	-3.76	-5.01
	(4.5)	(0.0)	-(3.5)	-(2.3)	-(3.7)	-(2.1)	-(7.5)	-(6.1)
Social Assistance	-6.30	-7.50	-10.00	-20.10	-15.10	-15.00	-10.00	-26.30
	-(22.7)	-(25.0)	-(27.6)	-(36.4)	-(30.2)	-(25.5)	-(20.0)	-(31.8)
Employment Insurance	8.80	8.80	8.80	16.30	12.80	16.30	7.50	10.00
	(31.8)	(29.2)	(24.1)	(29.5)	(25.5)	(27.7)	(15.0)	(12.1)
Unexplained	-6.30	-6.30	-5.00	-3.80	-4.30	-8.80	-3.80	8.80
	-(22.7)	-(20.8)	-(13.8)	-(6.8)	-(8.5)	-(14.9)	-(7.5)	(10.6)
B. Reverse order								
Sociodemographics	1.25	1.26	1.25	2.51	4.37	5.01	8.77	11.28
	(5.5)	(4.6)	(3.2)	(4.6)	(9.2)	(12.1)	(15.2)	(18.8)
Employment Insurance	5.01	6.26	7.52	8.76	8.84	10.02	11.27	13.78
	(22.2)	(22.7)	(19.4)	(15.9)	(18.5)	(24.2)	(19.6)	(22.9)
Social Assistance	-6.26	-6.26	-7.52	-8.76	-5.85	-1.25	-3.76	0.00
	-(27.8)	-(22.7)	-(19.4)	-(15.9)	-(12.3)	-(3.0)	-(6.5)	(0.0)
Old Age Security	6.30	6.30	7.50	8.80	6.00	1.30	5.00	1.30
	(27.8)	(22.7)	(19.4)	(15.9)	(12.6)	(3.0)	(8.7)	(2.1)
Child Benefits	0.00	1.30	5.00	10.00	8.40	8.80	11.30	13.80
	(0.0)	(4.5)	(12.9)	(18.2)	(17.7)	(21.2)	(19.6)	(22.9)
Unexplained	-3.76	-6.26	-10.02	-16.29	-14.17	-15.03	-17.53	-20.04
	-(16.7)	-(22.7)	-(25.8)	-(29.6)	-(29.7)	-(36.4)	-(30.4)	-(33.3)

Notes: Estimates are expressed in thousand dollars. The percentage change explained by factor is reported in parenthesis. The average income is greater than the median and is reported in column (5). Standard errors are available upon request; no reported because of space limitations. Author's calculation using the SLID 1996, 2006.

Table 6. Decomposing Differences in Disposable income Densities (1996-2006), reported by percentile.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Percentile	5	10	25	50	75	90	95
1996	11.51	15.17	21.35	31.35	43.48	56.90	68.12
2006	13.90	17.85	25.13	37.05	52.63	70.54	85.82
Total difference	2.39	2.69	3.78	5.70	9.15	13.65	17.70
Benchmark order							
Socioeconomics	1.33	1.41	1.96	3.19	5.10	8.03	10.24
	(3.9)	(3.4)	(2.8)	(1.7)	(2.9)	(5.3)	(7.2)
Child Benefits	3.74	4.12	6.80	7.86	10.05	10.81	12.95
	(11.1)	(10.0)	(9.6)	(4.2)	(5.8)	(7.2)	(9.1)
Old Age Security	0.00	0.00	-0.51	-0.71	-0.99	0.00	-1.54
	(0.0)	(0.0)	-(0.7)	-(0.4)	-(0.6)	(0.0)	-(1.1)
Social Assistance	-5.38	-5.47	-7.07	-5.95	-2.87	-1.28	-4.46
	-(16.0)	-(13.2)	-(10.0)	-(3.1)	-(1.7)	-(0.8)	-(3.1)
Employment Insurance	12.95	16.50	28.58	86.29	75.97	63.13	57.04
	(38.5)	(39.9)	(40.3)	(45.7)	(43.9)	(42.0)	(40.0)
Unexplained	-10.24	-13.88	-25.99	-84.98	-78.12	-67.04	-56.52
	-(30.5)	-(33.5)	-(36.6)	-(45.0)	-(45.1)	-(44.6)	-(39.6)

Notes: Estimates expressed in thousand dollars of 2010. Disposable income refers to the after-tax and transfers income. The percentage change explained by factor is reported in parenthesis. The average income is greater than the median and is reported in column (5). Standard errors are available upon request; no reported because of space limitations. Author's calculation using the SLID 1996, 2006.

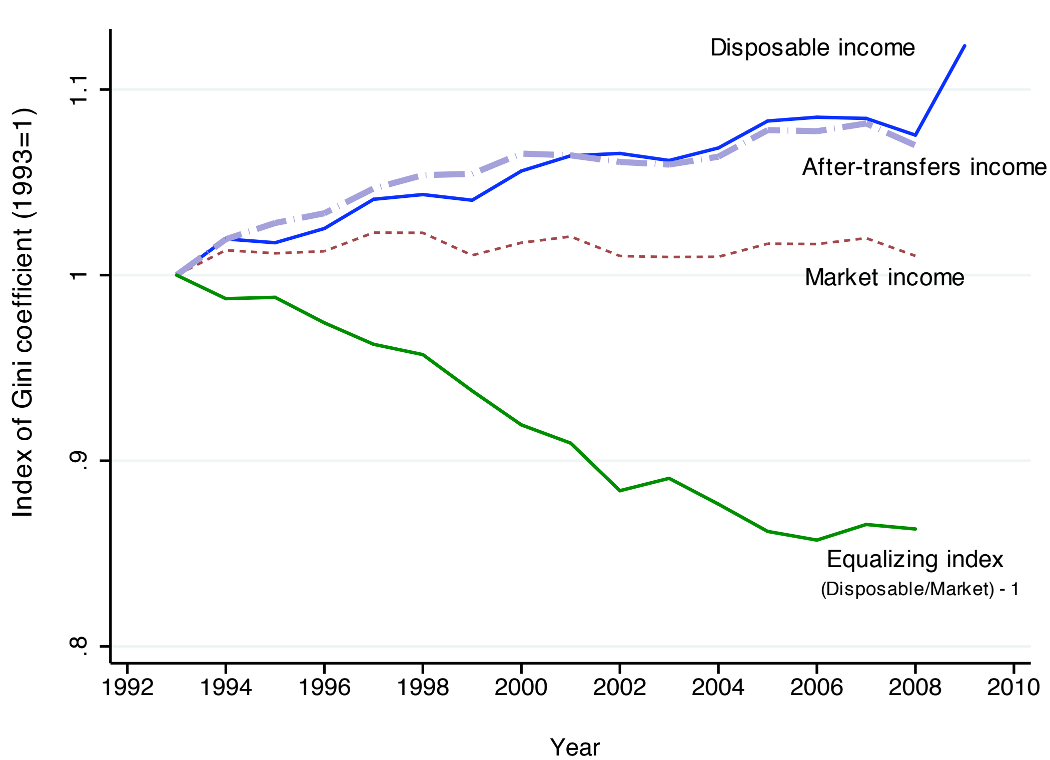

Figure 1. **Trends of Income Inequality: 1993-2009.** Inequality measured with the Gini coefficient and indexed to its 1993 value. Income refers to family equivalent units expressed in constant dollars of 2010. Market income refers to earnings, investment income, pension, alimony and other taxable income before taxes and transfers. After-transfers income is the sum of earnings and transfers. Disposable income is after-transfers income minus income tax. The equalizing index is defined in terms of the Gini coefficients: $\left({}^{Gini(y_{Disposable})}/_{Gini(y_{Market})} - 1\right) \times 100$. An equalizing index less than 1 indicates that the role of the tax and transfer system is declining relative to 1993.
Source: Author's estimates using the Survey of Labour and Income Dynamics, 1993-2009. Sample includes all families with after-transfer income greater than zero.

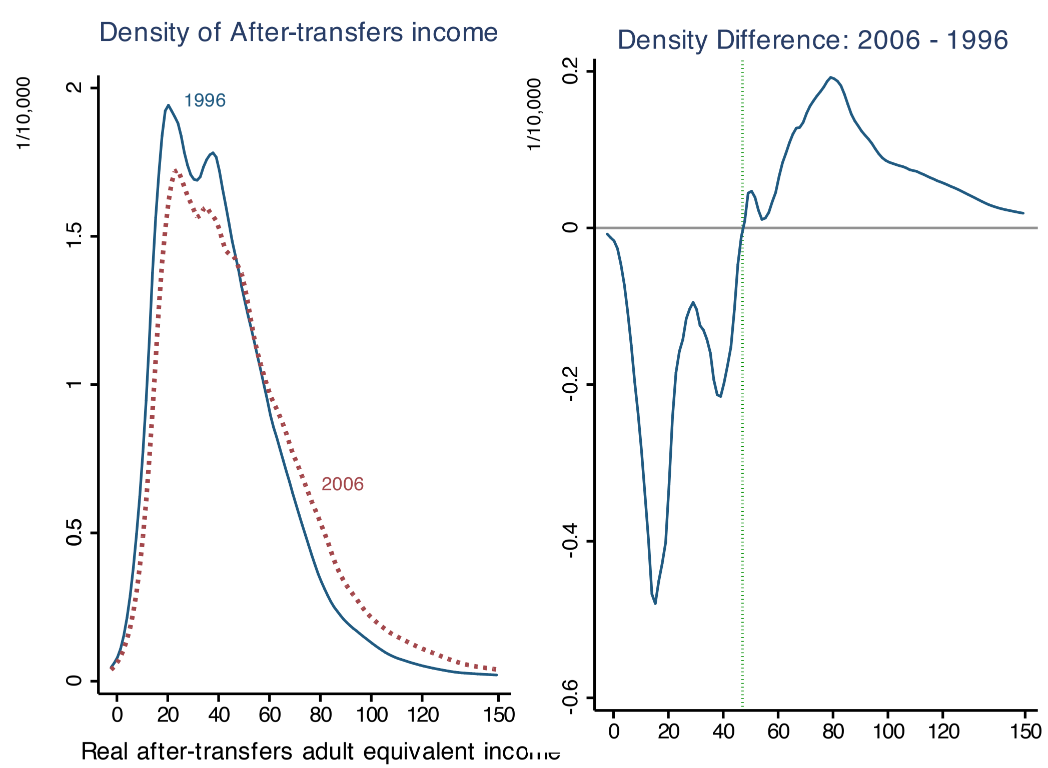

Figure 2. **Difference in After-transfer income density: 1996 - 2006.** After-transfer income is the sum of market income and government transfers, expressed in family equivalent units and in constant dollars of 2010. The density differences is obtained by subtracting density estimates of 1996 from that in 2006. The vertical line represents the income level where the density cross.

Source: Author's estimates using the Survey of Labour and Income Dynamics, 1996-2006. Sample includes main income earners and spouse of families with after-transfer income greater than zero.

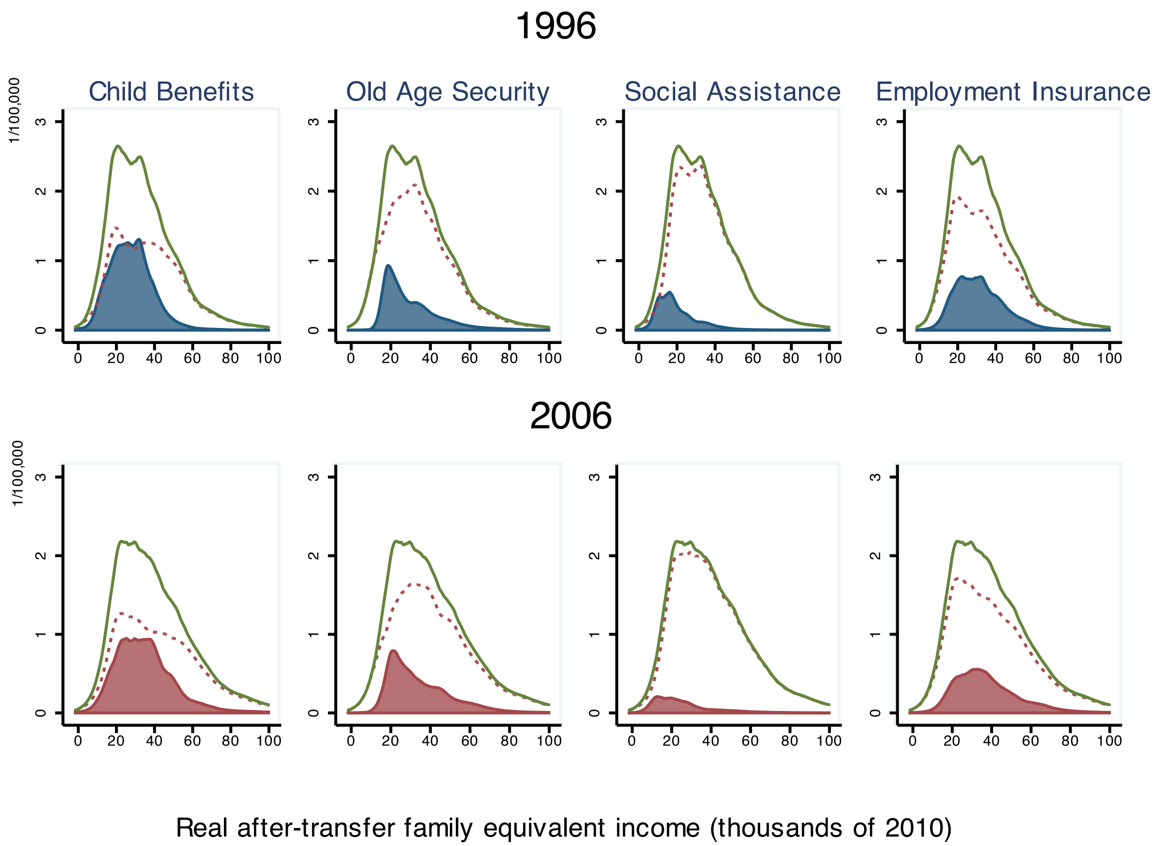

Figure 3. **Kernel density estimates of after-transfer income of families receiving program benefits: 1996 - 2006.** Income density of families with program benefits receipt (solid line), and income density of those who reported not receiving program benefits (dotted line). Income refers to family after-transfer (and before-tax) income expressed in family equivalent units and in constant dollars of 2010.

Source: Author's estimates using the Survey of Labour and Income Dynamics, 1996 and 2006. Sample includes main income earners and spouse of families with after-transfer income greater than zero.

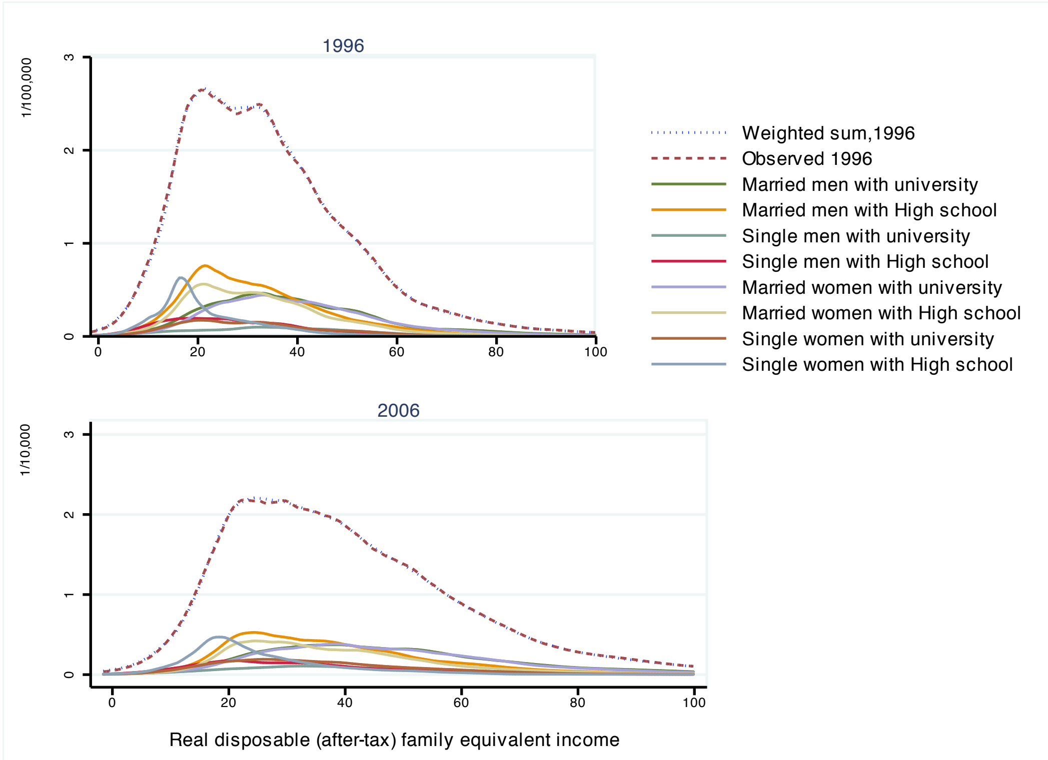

Figure 4. Income densities by eight socioeconomic groups defined interacting three binary variables: sex, marital status summarized by married or in common law, and education attainment summarize by high school or university graduate. The weighted sum of cell sub-densities adds to the total income density in 1996 and 2006. Author's estimates using the SLID of 1996 and 2006.

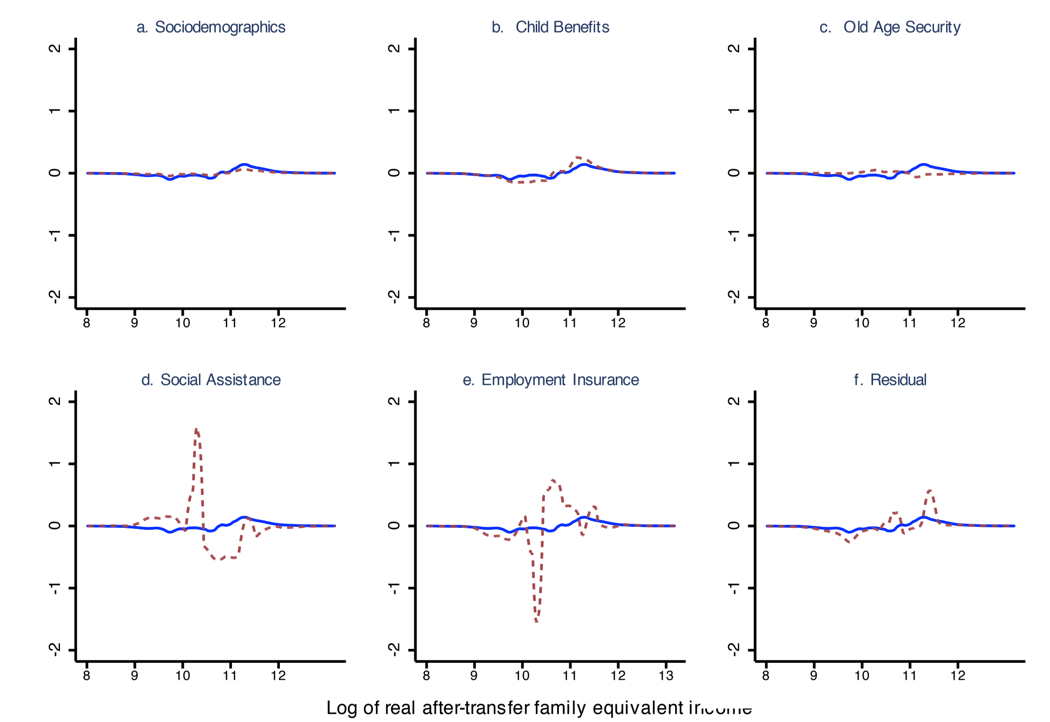

Figure 5. **Marginal effects of decomposition factors: differences between adjusted (reweighted) 2006 density and the actual density in 2006.** Log of real after-transfer family equivalent income in prices of 2010. The solid line in all panels is the total difference to be explained: 2006-1996. The dashed line is the difference between the actual and counterfactual densities. These differences can be interpreted as the *marginal effects* of a given explanatory factor, which vary over the entire distribution. Source: Autor's estimates using the Survey of Labour and Income Dynamics. Sample includes main income earners and spouse of families with after-transfer income greater than zero.

B Appendix

Let y denote income and x is a vector of socioeconomic characteristics that include program eligibility requirements. τ^j and τ^k are binary variables indicating whether a family received benefits from program j (for instance Child Benefits) or benefits from program k (for instance Social Assistance).

The conditional distribution of income in 2006 can be expressed as the integral over the conditional (on observed characteristics) income density, $f_t(y \mid x)$, over the marginal distribution of the observed characteristics at date t, $dF_x(x)$ over the domain set of the integrand Ω_x:

$$
\begin{aligned}
f_{06}(y_{06}) &= \int\int\int\int f_{06}(y_{06}, x_{06}, \tau^j_{06}, \tau^k_{06}) \cdot dF(y) dF(x_{06}) dF(\tau^j_{06}) dF(\tau^k_{06}) \\
&= \int_{\Omega_x} f_{06}(y_{06}, \tau^j_{06}, \tau^k_{06} | x_{06}) \cdot dF(x_{06}) dx \\
&= \int_{\Omega_x}\int_{\Omega_{\tau^j}} f_{06}(y_{06}, \tau^k_{06} | x_{06}, \tau^j_{06}) \cdot dF(x_{06}) dF(\tau^j | x; t_{\tau_j|x} = 06) dx d\tau_j
\end{aligned}
$$

Let A be the decomposition sequence of factors: X, τ^j and τ^k. Let B be an alternate ordering of the decomposition: X, τ^k, τ^j. The counterfactual density identifying the changes in the distribution explained by program j in the decomposition sequence A is defined by:

$$
\begin{aligned}
f_{06}^{C_{j|x}}(y_{06}) &= \int_{\Omega_x}\int_{\Omega_{\tau^j}} f_{06}(y_{06}, \tau^k_{06} | x_{06}, \tau^j_{06}) \cdot dF(x_{96}) dF(\tau^j_{96} | x_{96}) dx d\tau^j \\
&= \int_{\Omega_x}\int_{\Omega_{\tau^j}} f_{06}(y_{06}, \tau^k_{06} | x_{06}, \tau^j_{06}) \cdot \psi_x(x) \psi_{j|x}(j|x) \cdot dF(x_{06}) dF(\tau^j_{06} | x_{2006}) dx d\tau^j
\end{aligned}
$$

where $\psi_{j|x}(j|x) = \frac{dF(\tau^j_{96}|x_{96})}{dF(\tau^j_{06}|x_{06})}$ is the reweighting function generated as the propensity score function of receiving benefits from program j conditional on characteristics x; which captures the change in the probability of receiving benefits τ^j for a family with the same characteristics from before and after the program reforms.

The third factor of sequence A explains changes associated to program k once we accounted for changes in socio-demograhics x and the receipt of program τ^j:

$$
\begin{aligned}
f_{06}^{C_{k|j,x}}(y_{06}) &= \int_{\Omega_x}\int_{\Omega_{\tau^j}}\int_{\Omega_{\tau^k}} f_{06}(y_{06} | x_{06}, \tau^j_{06}, \tau^k_{06}) \cdot dF(x_{96}) dF(\tau^j_{96} | x_{96}) \cdot dF(\tau^k_{96} | \tau^j_{96}, x_{96}) dx d\tau^j d\tau^k \\
&= \int_{\Omega_x}\int_{\Omega_{\tau^j}}\int_{\Omega_{\tau^k}} f_{06}(y_{06} \mid x_{06}, \tau^j_{06}, \tau^k_{06}) \cdot \psi_x(x) \psi_{j|x}(j|x) \psi_{k|j,x}(k|j,x) \cdot dF(x_{06}) dF(\tau^j_{06} | x_{06}) \\
&\qquad \cdot dF(\tau^k_{06} | \tau^j_{06}, x_{06}) dx d\tau^j d\tau^k
\end{aligned}
$$

To compare the results generated under the ordering A with those in ordering B, the analogue of the counterfactual of τ^k in ordering B where τ^j is the second in the sequence is defined by:

Note that a counterfactual density can be interpreted as a functional transformation of the observed density in 2006, since $f_{06}^{C_{\tau_j,x}}(y_{06}) \approx \psi_{j|x}(j|x) \cdot f_{06}(y_{06})$. Consequently, the total decomposition of difference in the density of 1996 and 2006 explained by these three factors, x, τ^j and τ^k can be expressed, under ordering option A by:

$$
\begin{aligned}
f_{06}(y_{06}) - f_{96}(y_{96}) \approx\ & f_{06}(y_{06}) - \psi_x(x) \cdot f_{06}(y_{06}) + \psi_x(x) \cdot f_{06}(y_{06}) - \psi_{j|x}(j|x) \cdot f_{06}(y_{06}) + \\
& \psi_{j|x}(j|x) \cdot f_{06}(y_{06}) - \psi_{k|j,x}(k|j,x) \cdot f_{06}(y_{06}) + \psi_{k|j,x}(k|j,x) \cdot f_{06}(y_{06}) - f_{96}(y_{96})
\end{aligned}
$$

The akin expression for the sequence B is similarly obtained. From comparing the expressions of decomposing total differences from 1996 to 2006 using two alternative sequence of factors A and B, we obtain the condition for a decomposition to be order invariant.

A decomposition is order invariant if ordering A and ordering B yield the same results by factor, which is equivalent to the expression in proposition (1) to be satisfied:

$$\psi_x(x) \cdot \psi_{j|x}(j|x) \cdot \psi_{k|j,x}(k|j,x) = \psi_{k|x}(k|x) \cdot \psi_{j|k,x}(j|k,x)$$

condition that can be expressed by:

$$\frac{dF(\tau_{96}^k|x_{96})}{dF(\tau_{06}^k|x_{06})} \cdot \frac{dF(\tau_{96}^j|\tau_{96}^k, x_{96})}{dF(\tau_{06}^j|\tau_{06}^k, x_{06})} = \frac{dF(\tau_{96}^j|x_{96})}{dF(\tau_{06}^j|x_{06})} \cdot \frac{dF(\tau_{96}^k|\tau_{96}^j, x_{96})}{dF(\tau_{06}^k|\tau_{06}^j, x_{06})}$$

the non order invariant condition is satisfied when decomposition factors τ^j and τ^k are independent. ■