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Fiscal Policy and Default Risk in Emerging Markets

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Abstract
Emerging economies usually experience procyclical public expenditures, tax rates and private consumption, countercyclical default risk, interest rate spreads and current account and higher volatility in consumption than in output. In this article we develop a dynamic stochastic equilibrium model of a small open economy with endogenous fiscal policy, endogenous default risk and country interest rate spreads in an incomplete credit markets framework that rationalizes these empirical findings.

Keywords: Procyclical fiscal policy, Sovereign default.

JEL Classification: F34,F41

Resumen
Las economías emergentes usualmente presentan gasto público, tasas de impuestos y consumo privado procíclicos, riesgo de default, tasas de interés y cuenta corriente contracíclicos y volatilidad del consumo mayor que la del producto. En este artículo se desarrolla un modelo dinámico y estocástico de una economía pequeña y abierta con política fiscal endógena, tasas de interés y riesgo de default endógenos en un contexto de mercados incompletos que racionaliza estas regularidades empíricas.

Palabras Clave: Política fiscal procíclica, Default soberano.

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1. Introduction

Developing countries differ from developed countries in different aspects. In this paper we focus on the behavior of fiscal variables, sovereign interest rate spreads and default risk in emerging market economies. Whereas in most middle and low income economies government spending appears to be highly procyclical, i.e., government spending rises in economic expansions and falls in recessions, in industrialized countries it usually presents an acyclical behavior. Evidence on the cyclical properties of the fiscal position suggests that it is procyclical in developing economies. Additionally, the inflation tax tends to accelerate in recessions, which is associated with financial crises. In advanced economies the opposite happens, the inflation tax increases when output growth is high. Therefore, whereas in the developing world fiscal policy has been procyclical, the opposite holds in developed countries. A procyclical fiscal policy implies higher (lower) public spending and lower (higher) tax rates in good (bad) times.

Developing economies also have more volatile business cycles and are more crisis prone than developed economies. In the last two decades they have experienced several episodes of sovereign default, some of the most recent cases being Ecuador in 1999, Argentina in 2001 and Uruguay in 2002 among others. Sovereign default usually takes place in bad times when output growth is low or even negative. In addition, these countries face countercyclical interest rates in international credit markets (external credit is more expensive in bad times) which is tightly linked to countercyclical default risk.

In this paper we rationalize these stylized facts on fiscal variables, interest rates and default risk of emerging market economies documented in the empirical literature.
and we analyze the links between these variables by developing a quantitative dynamic stochastic model of a small open economy with endogenous fiscal policy, endogenous borrowing constraints and endogenous default risk.

In the model economy there are households, a domestic government and foreign lenders. Households value private consumption, public spending and leisure. Public spending provides direct utility to the private sector, which prefer a smooth path of public spending over a volatile one. The government collects consumption taxes from households and borrows abroad to finance public expenditures. Markets are incomplete since the only asset available to the government is a non-contingent one period bond. Sovereign debt contracts are unenforceable, the government has the option to default on the outstanding debt every period. Foreign lenders charge a risk premium that accounts for the default risk they face, hence interest rate spreads reflect the sovereign default risk. Since the repayment of non-contingent loans is more costly in recessions, the incentives to default are higher in bad times. Thus, in recessions the government faces higher interest rate spreads due to higher default risk and finds it optimal to rely more heavily on taxation to finance public expenditures. However, in expansions the marginal cost of international credit is lower so there is an increase in financing through borrowing, while taxes play a lesser role. Thus, tax rates are procyclical. Additionally, since the one period non-contingent bond is not a good instrument for consumption smoothing purposes, the government is not able to smooth public spending, so public expenditures are highly procyclical. Therefore, the government optimally implements a procyclical fiscal policy.

In this sense, our paper studies the dynamic interaction between endogenous fiscal policy and other key macroeconomic variables such as sovereign debt financing in
emerging economies. The dynamics of taxation, public expenditures, default, interest rate spreads and international capital flows in the model are derived as an equilibrium result from the interaction between the domestic government, the private sector of the small open economy and lenders in international credit markets.

In a quantitative analysis, we calibrate the model to the Argentinian economy\(^1\). Results show that the calibrated model mimics all the empirical regularities described above for emerging economies.

The paper proceeds as follows: Section 2 provides the link to the literature; the economic environment and the theoretical model are presented in Section 3, the equilibrium is defined in Section 4, the quantitative implications of the model are analyzed in Section 5 and the conclusions are presented in Section 6. The algorithm is described in the appendix.

2. Link to the Literature

The empirical regularities for emerging economies addressed in this paper are well documented in the literature. Gavin and Perotti (1997) (from now on GP) find that fiscal policy is procyclical in Latin America, i.e., that it tends to be expansionary in good times and contractionary in bad times. Talvi and Vegh (2000) (from now on TV) argue that far from being a particular feature of Latin American countries, procyclical fiscal policy appears to be the norm among developing economies. More recently, a study by Kamisky, Reinhart and Vegh (2004) (from now on KRV) reviews the empirical evidence on the procyclicality of macroeconomic policy analyzing countries grouped by

\(^1\)In December 2001, Argentina defaulted on its foreign debt and fell into a deep economic crises. This is the major case of sovereign default in recent history. Thus, most of the recent quantitative papers of sovereign default focus on Argentina.
income levels. They find that OECD countries seem to implement either countercyclical or acyclical fiscal policies. However, consistently with previous studies, they observe that fiscal policy appears to be predominantly procyclical in low and middle income countries.

TV are among the first to document that public spending is highly procyclical in the developing world and acyclical in the G-7 economies. Subsequent empirical studies (Braun (2001), Lane (2003), and others) find similar evidence supporting the procyclical behavior of government expenditures in developing countries.

GP observe that while governments in developed economies rely mostly on direct taxation for revenues, Latin American countries depend heavily on non tax revenues and indirect taxes (including taxes on international trade), i.e., taxes on goods and services. Indirect taxes account on average for almost 55% of total tax revenues in the region and show an upward trend, and international trade taxes account for 16%.

There is no readily available data on the behavior of tax rates over the business cycle. However, casual evidence from developing countries strongly suggest that tax rates are procyclical in these economies. Consider for example the cases of Mexico and Argentina in 1995. These economies were in the middle of a recession and both governments implemented a contractionary fiscal policy. Tax rates were increased and public expenditures were reduced in both countries. In addition, GP argue that inflation has had a close link with fiscal policy in Latin America. While acknowledging that inflation may have been in part the endogenous result of financial crises associated with large fiscal imbalances, they point out that fiscal deficits in Latin American economies have been associated with subsequent increases in inflation. This suggests the presence of fiscal dominance since inflation has acted much more like an instrument of fiscal policy.
in these countries compared to developed economies. Thus, in this context they analyze the cyclical properties of the inflation tax. GP find that the inflation tax is low in good times and high in bad times. In sharp contrast to the evidence for developing economies, Cooley and Hansen (1995) have found that in advanced economies the inflation tax tends to increase during expansions and fall during recessions.

The documented procyclicality of fiscal policy in most developing countries is in opposition with the normative fiscal policy prescriptions. Keynesian models imply that fiscal policy should be countercyclical. In good times, the government should reduce public expenditures and increase tax rates. In bad times the government should do the opposite. In contrast, neoclassical models of fiscal policy inspired in Barro (1979) implies that fiscal policy should remain basically neutral over the business cycle. Therefore, if governments followed the Keynesian prescription, fiscal policy would be countercyclical. On the other hand, if governments followed the prescription from tax smoothing models, fiscal policy would be acyclical.

What explains the procyclical character of fiscal policy in most developing countries? Several explanations have been advanced in order to explain the procyclicality of fiscal policy, including weak political institutions, incomplete markets and borrowing constraints.

Lane and Tornell (1999) argue that in economies without strong legal and political institutions a "voracity" effect may take place. In good times a windfall in fiscal revenues intensifies the struggle for public resources. Every interest group tries to get its share without completely internalizing the effect on general taxation. As a result, a more than proportional rise in public spending is observed. Talvi and Vegh (2000) develop a model that includes a political distortion, which increases the cost of running fiscal
surpluses during economic expansions. In this model, the government cuts tax rates in
good times in order to avoid spending pressures. Alesina and Tabellini (2006) explain
the procyclicality of fiscal policy with a political agency model. Voters observe output
accurately but not the rents appropriated by corrupt governments. In good times, voters
demand more public goods or lower taxes as they anticipate that, otherwise, windfall
revenues will be wasted by corrupt governments.

Riascos and Vegh (2003) develop a neoclassical model of fiscal policy in which public
consumption provides direct utility to households. The government optimally chooses
both the level of public consumption and the tax rate. They show that, with com-
plete markets government consumption is acyclical. They also consider the incomplete
markets case, where the only asset available to the economy is risk-free debt. With
incomplete markets the government is less able to smooth its consumption and the cor-
relation between output and public consumption becomes positive. They suggest that
the lack of complete markets in developing countries might be an important determi-
nant of the cyclical properties of government spending in these economies. However, the
government can commit to pay its debt, so it faces the same interest rate across states.
Since the government always borrows at the international risk free rate, the model is
not able to generate a negative correlation between output and tax rates.

Besides documenting the cyclical properties of fiscal variables in Latin American
economies, GP stress the role of borrowing constraints in explaining the procyclicality
of fiscal policy in these countries. They argue that governments in Latin America face
a loss of access to external credit markets during bad times, which makes it extremely
difficult to implement either a countercyclical or an acyclical fiscal policy. GP present
some evidence that international credit constraints are more severe during recessions.
Although these constraints cannot be observed directly, they infer their presence from the use of IMF emergency credit, which is usually provided when there are no other sources of financing. They find that the use of these credits is more frequent in bad macroeconomic times. However, GP only emphasize the role of borrowing constraints without developing a theoretical model.

Aizenman, Gavin and Hausmann (2000) construct a two period model in which the government can either tax households or borrow from abroad to finance an exogenous amount of public goods. The government can default on its debt and a large recession forces it to its credit ceiling, resulting in a complete loss of market access, in which case the government is forced to increase taxes to finance its expenditures.

This paper considers a quantitative dynamic stochastic small open economy model with endogenous fiscal policy and default. The paper is specially related to the recent literature of quantitative models of sovereign debt. Following the seminal theoretical model on international lending and sovereign default by Eaton and Gersowitz (1981) and the more recent quantitative analysis on unsecured consumer default by Chaterjee, Rios-Rull et.al. (2002), quantitative models on sovereign default starting by Aguiar and Gopinath (2006) and Arellano (2006) and followed by Cuadra and Sapriza (2006), Lizarazo (2006) and Yue (2006) among others have analyzed different features of the dynamics of emerging market sovereign debt crises. However, these studies do not analyze fiscal policy.

As far as we know, this paper presents the first effort to explain the procyclical behavior of fiscal policy in developing countries using a quantitative model of sovereign debt and default in an incomplete markets framework. In addition, the model helps to clarify several aspects of the explanation based on the tightening of borrowing constraints in
bad times. Some authors have interpreted it as a full cut off from international credit markets during recessions. In this sense, TV criticize this explanation arguing that developing countries do not always lose access to foreign borrowing during recessions. In our paper the government only defaults when the recession is sufficiently large and the government is already highly indebted. In such scenario, the country is temporarily excluded from external markets and public expenditures have to rely on taxes. It is important to notice that although defaulting is more tempting in bad times, an adverse shock does not always induce the government to default on its foreign debt. When the government faces a negative shock and does not default, it has the option to borrow from abroad, though at a high interest rate. Since the cost of external loans increases in recessions, financing public expenditures with foreign debt becomes a relatively less attractive option. Thus, the possibility of default, and the associated risk premium in an incomplete markets setting, is the major factor that induces a procyclical fiscal policy. In this context, the countercyclical behavior of interest rates in the model is consistent with the empirical evidence from emerging markets. Neumeyer and Perri (2005) and Uribe and Yue (2004) have documented that the cost of foreign credit is higher in recessions than in expansions. The fact that sovereign spreads are higher in bad macroeconomic times is closely linked to the default probabilities that foreign lenders perceive from these economies. Empirical studies have estimated these probabilities, finding that incentives to default are higher in recessions. Cantor and Packer (1996) have found that sovereign credit ratings, which are valuations on the probability that a borrower will pay back its debts, strongly respond to the GDP growth rate.

\footnote{See Peter (2002) for a survey on econometric studies of the probability of sovereign default.}
3. The Model

We consider a small open economy model with three agents: households, government and foreign lenders. Households have preferences over private consumption, public spending and leisure. They work and consume, taking as given what the government does. Households produce goods using labor as an input and the production function is subject to technology shocks. The government cares about households and seeks to maximize their utility. It borrows from abroad, taxes households and finances public expenditures. Markets are incomplete since the only asset traded in international credit markets is a one period non contingent bond that is available to the government\(^3\), which is the only domestic agent that is able to borrow and lend\(^4\). Debt contracts are not enforceable since the government has the option to default on them. When it defaults, it is temporarily cut off from credit markets and the economy suffers an output loss. Foreign lenders charge a premium to account for the probability of not being paid back by the government.

3.1. Households

There is a representative household with preferences given by the present value of the sum of instantaneous utility functions:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, G_t, 1 - l_t)
\]  

(3.1)

The per period utility is concave, strictly increasing and twice differentiable. The dis-

\(^3\)This assumption is not too strong given the fact that emerging markets usually have depended on short term financial instruments.

\(^4\)Most external debt in developing countries represents government debt. For example, as of March 2003, government debt accounts for almost 70% of the total stock of foreign debt in Argentina.
count factor is $\beta \in (0, 1)$ and households derive utility from private consumption, public expenditures and leisure. Let $C_t$ represents private consumption, $G_t$ public spending and $l_t$ the amount of time allocated to produce goods. Output is produced using labor services and the production technology is subject to productivity shocks.

$$y_t = A_tF(l_t)$$

where $A_t$ represents the productivity factor which is assumed to follow a Markov process, with $Q(A_{t+1}|A_t)$ denoting the Markov transition function for $A$. Thus, the productivity factor assumes discrete values defined over the set $\Upsilon$. Output can be divided between private and public consumption.

The government implements a tax on private consumption\(^5\) and it has two instruments to finance its expenditures: the proceeds from taxation and external borrowing. The representative household takes public expenditures and taxation as given and makes private consumption and labor decisions, subject to the following budget constraint:

$$(1 + T_t)C_t = A_tF(l_t) \quad (3.2)$$

where $T$ is the tax rate on private consumption.

From households’ first order conditions we get the following equation, which describes households’ optimal behavior.

$$\frac{U_t(C, G, 1 - l)}{U_c(C, G, 1 - l)} = \frac{AF_t(l)}{(1 + T)}$$

\(^5\)As mentioned before, GP observe that Latin American countries depend heavily on non tax revenues and indirect taxes, i.e., taxes on goods and services.
3.2. Government

The benevolent government maximizes the utility of the households in the economy. The government can participate in international credit markets where it can lend and borrow. Financial markets are incomplete since the government can only save and indebt itself by selling and buying a non contingent one period bond. In order to finance its expenditures, the government has two options, it can either borrow from abroad or tax households through a consumption tax.

Each period, the government has the option to choose between honoring its outstanding foreign debt or defaulting on it. This decision comes from comparing the net benefits of the two options, the government makes an optimal comparison between the cost of temporary exclusion from credit markets given by the foregone benefits of consumption smoothing and the output loss in autarky, against the direct costs of repayment given by the short-run disutility of repaying the non contingent loan.

The intertemporal problem of the government is expressed in a recursive dynamic programming form. Conditional on having access to credit markets, the government has to decide whether to default or not; if default is not optimal then it has to decide how much borrowing or saving to do and its fiscal policy, i.e., the amount of public expenditures and the level of the consumption tax. If default is optimal then the government only has to decide its fiscal policy. All these decisions are made given the productivity shock and the amount of outstanding foreign assets it has. Thus, the state variables are the productivity factor $A$, the level of foreign assets $B$ and the credit situation of the country $d$, where $d = 1$ if the economy has access to credit markets and 0 if the country is in financial autarky.

The value function when the government has access to international markets and
begins the period with an amount of assets $B$ and shock $A$ is given by $V_0(B, A)$. The government has to decide between honoring its debt and defaulting on it, by comparing the value associated with paying back and remaining in the credit market $V^c(B, A)$, with the value associated with defaulting and going to temporary autarky $V^d(A)$. The problem can be expressed in the following way:

$$V_0(B, A) = \max \left\{ V^c(B, A), V^d(A) \right\}$$  \hspace{1cm} (3.3)

and the optimal default decision of the government is characterized by

$$D(B, A) = \begin{cases} 
1 & \text{if } V^c(B, A) > V^d(A) \\
0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (3.4)

The default policies determine a repayment set $\Gamma(B)$; this is defined as the set of values of the productivity shock such that repayment is the optimal decision given the level of foreign assets $B$,

$$\Gamma(B) = \{ A \in \Upsilon : D(B, A) = 1 \}$$  \hspace{1cm} (3.5)

and a default set $F(B)$ defined as the set of values of the productivity shock such that default is optimal given asset holding level $B$,

$$F(B) = \{ A \in \Upsilon : D(B, A) = 0 \}$$  \hspace{1cm} (3.6)

If the government does not default, it can issue new debt and finance its expenditures according to the following restriction:

$$G = TC + B - q(B', A)B'$$  \hspace{1cm} (3.7)

where $q(B', A)$ is the price of the bond that pays one unit of consumption goods the following period if the government does not default on its debt. When the government
borrows, it sells bonds to foreign lenders, this transaction implies that the government receives \( q(B', A)B' \) units of consumption goods from foreign creditors on the current period and promises to pay \( B' \) units next period conditional on not defaulting.

In a similar way, when the government lends, it buys bonds from foreign creditors. If the government purchases a bond of value \( B' \), this transaction implies that the government lends \( q(B', A)B' \) units of the good to foreign creditors and it will receive \( B' \) units the following period. When foreign creditors borrow from the domestic government it is assumed that they always pay back, thus only the government is not able to commit to repay its debt. Hence when the government issues bonds, the price of them reflects the probability of default, so the price depends on \( B' \) and on \( A \) since the incentives to default depend on these factors.

The government’s decision problem when it has access to credit markets is to choose the tax rate, public consumption and foreign assets in order to maximize households’ utility, taking into account how the private sector will respond to these policies. Formally, the government maximizes utility subject to households’ budget constraint and the private sector’s first order conditions, as well as its own budget constraint.

Thus, the problem of the government when it has access to credit markets is:

\[
V^c(B, A) = \max_{T, G, B'} \left\{ U(C^*, G, 1 - l^*) + \beta \sum_{A'} V_0(B', A')Q(A'/A) \right\}
\]

\[\text{s.t.}\]

\[
G = TC^* + B - q(B', A)B'
\]

\[
AF\left(l^*\right) = (1 + T)C^*
\]

\[
AF_i\left(l^*\right) = \frac{U_i\left(C^*, G, 1 - l^*\right)}{U_c\left(C^*, G, 1 - l^*\right)}
\]
where \( C^\ast \) and \( l^\ast \) represent the optimal choice of private consumption and labor effort made by households, given the government’s policies (i.e., \( C^\ast \) and \( l^\ast \) solve the households’ first order conditions and budget constraint, given the policies implemented by the government: \( T, G, \) and \( B' \)).

When the government defaults on its debt the country is temporarily excluded from international credit markets\(^6\). In addition, the economy suffers an output loss\(^7\), so that with the same amount of labor services households produce less goods. The productivity factor in autarky is represented by \( h(A) \). The problem is:

\[
V^d(A) = \max_{T_d,G_d} \left\{ U(C^\ast_d, G_d, 1 - l^\ast_d) + \beta \sum_{A'} \left[ \mu V_0(0, A') + (1 - \mu)V^d(A') \right] Q(A'/A) \right\}
\]

s.t.

\[
G_d = T_d C^\ast_d \\
h(A) F_l(l^\ast_d) = (1 + T_d) C^\ast_d \\
\frac{h(A) F_l(l^\ast_d)}{(1 + T_d)} = \frac{U_l(C^\ast_d, G_d, 1 - l^\ast_d)}{U_c(C^\ast_d, G_d, 1 - l^\ast_d)}
\]

\( C^\ast_d \) and \( l^\ast_d \) represents the optimal consumption and labor decisions when the country is in autarky. The tax on consumption is the only instrument to finance public expenditures.

If the economy is excluded from credit markets, in such a case the following period the country may regain access to external markets, where \( \mu \) denotes the exogenous probability of reentering financial markets. When the economy returns to financial markets, it does so with no debt burden, \( B = 0 \), so foreign lenders forgive debt for the

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\(^6\)When the Government defaults, the country goes to autarky. However, the following period it returns to credit markets with a positive probability. The probability of redemption is calibrated such that the country stays in autarky three years on average.

\(^7\)The assumption that default reduces output can be rationalized by the common view that after a default episode there is a disruption in foreign trade, Rose (2003), which induces an output loss.
domestic government. In this case the continuation value for party 1 will be $V_0(0, A')$. On the other hand, with a probability $1 - \mu$ the country will stay in autarky. In such a case, the continuation value for party 1 will be $V^{d}(A')$.

### 3.3. Foreign Lenders

There is a large number of identical, infinitely lived foreign creditors. Each lender can borrow or lend resources at the risk free rate $r_f$ and participates in a perfectly competitive market to the small open economy.

The individual lender is risk neutral and maximizes expected profits, which are given by the following equation

$$\Phi = -qB' + \frac{\lambda(B', A)}{1 + r_f}0 + \frac{(1 - \lambda(B', A))}{1 + r_f}B'$$

As it was pointed out by Cole and Kehoe (1996), the assumption of risk neutrality of lenders captures the idea that the analysis considers that compared to international credit markets, the domestic economy is small.

The first term of the equation above shows that when creditors lend to the government in the current period, they buy the discount bond issued by the domestic government at a price $q$. Next period the lenders may receive the face value of the bond depending on whether the government defaults or not. When it defaults, creditors get 0 units of the consumption good, where $\lambda(B', A)$ is the endogenous probability that the government defaults on its sovereign debt. Therefore, with probability $1 - \lambda(B', A)$ lenders will be paid back an amount $B'$.

Since there is perfect competition in the credit market, a zero profit condition for the foreign creditor is satisfied.
The bond price is then:
\[ q(B', A) = \frac{(1 - \lambda(B', A))}{1 + r_f} \] (3.10)

Thus, the equilibrium bond price \( q(B', A) \) reflects the probability of default of the government, \( \lambda(B', A) \), which results from
\[ \lambda(B', A) = \sum_{A' \in F(B')} Q(A'/A). \] (3.11)

so that the default probability is zero when \( F(B') = \emptyset \) and it is one when \( F(B') = \Upsilon \).

4. Equilibrium

4.1. Definition

In equilibrium households choose optimal private consumption and labor effort given government policies and the government determines its optimal default policy and asset holding policy as well as its optimal fiscal policy subject to the private sector optimizing and foreign lenders optimizing by satisfying their zero profit condition from the debt contract.

**Definition 4.1.** A recursive equilibrium for this small open economy is characterized by

1. A set of value functions \( V_0, V^c \) and \( V^d \) for the government,

2. A set of policy functions for household’s consumption \( C^*, C^*_d \) and household’s labor supply \( l^*, l^*_d \).

Alternatively, we could assume that foreign lenders have access to two instruments: a risky bond and a risk free bond. Since creditors are risk neutral, they are willing to buy the risky asset as long as its expected return equals the return of the risk free asset: \( (1 - \lambda)(1 + R) = 1 + r_f \), with \( q = \frac{1}{(1 + R)} = \frac{(1 - \lambda)}{1 + r_f} \).

4. A bond price function $q$,

such that

1. Given the government policies and the bond price function, the household policies for consumption and labor solve the household’s problem,

2. Given the bond price function $q$ and the optimal policies for households, the government’s value functions $V_0, V^c$ and $V^d$ and its policy functions $D, B', G$ and $T$ solve 3.3, 3.8 and 3.9:

3. The equilibrium bond price $q(B', s)$ is such that all agents in the small open economy are optimizing and international lenders get zero expected profits.

4.2. First Order Conditions from Government’s Problem

The first order condition for the consumption tax rate simplifies to:

$$U_c(C, G, 1 - l) \frac{AF(l^*)}{(1 + T)^2} = U_g(C, G, 1 - l) \left[ \frac{AF(l^*)}{(1 + T)^2} + \frac{TAF_l(l^*)}{1 + T} \frac{\partial l}{\partial T} \right].$$

This condition can be interpreted in terms of marginal benefits and marginal costs of changing the tax rate. In terms of effects on today’s utility, a marginal increase in the tax rate affects private consumption, public spending and labor effort. The government increases taxes and reduces the consumption of private goods in $\frac{AF(l^*)}{1 + T}$ units, which would reduce today’s welfare. On the other hand, the government can
increase public spending in \( \frac{AF(t^*)}{(1+T)^2} + \frac{TAF_i(t^*)}{(1+T)} \frac{\partial l}{\partial T} \) units, increasing household’s welfare.

The first term in this expression corresponds to the resources reallocated from private consumption to public consumption. However, if households react working less hours when the government implements a higher tax rate, then the amount of resources that can be assigned to public expenditures would be reduced. This effect corresponds to the second term.

The Euler equation for the government is obtained from the first order condition of its dynamic optimization problem and the envelope theorem:

\[
U_g(C, G, 1 - l) \left[ q + B' \frac{\partial q}{\partial B'} - \frac{TAF_i(t^*)}{(1+T)} \frac{\partial l}{\partial B'} \right] = \beta \sum_{A' \in \Gamma(B)} U_g(C', G', 1 - l') Q(A'/A)
\]

The Euler equation is interpreted in terms of marginal benefits and marginal costs of additional lending or borrowing. We consider the case where the government is a net debtor and the marginal benefits and costs can be described as follows:

In terms of effects on current welfare, for each unit of additional borrowing the government could increase the level of public spending in \( q + B' \frac{\partial q}{\partial B'} - \frac{TAF_i(t^*)}{(1+T)} \frac{\partial l}{\partial B'} \) units. Borrowing from international capital markets implies issuing bonds and additional borrowing would affect the amount of resources available to finance public spending. Since the government is borrowing more it could afford a higher level of public expenditures, in principle it could increase public consumption in \( q \) units, but at the same time if the government is already indebted, additional borrowing would increase the interest rate that it faces in international markets. As it was explained before, default probabilities are increasing in foreign debt, so if the government wants to borrow more, foreign lenders would require a higher risk premium. The decrease in the bond price implies
that the government is not getting as many resources from selling bonds as it would get if the bond price were constant. Therefore, when evaluating the benefits and costs of additional external borrowing, the government takes into account the fact that issuing more bonds would decrease the price, this effect correspond to the term $B' \frac{\partial q}{\partial B'}$. Although the country is small, the government’s borrowing can affect the idiosyncratic bond price that it faces: if the government issues additional bonds, default probabilities will be higher and foreign creditors will demand a lower price, otherwise they will not buy the bonds. In addition, extra borrowing might affect the labor supply and the level of output. If households responded supplying less labor then, for a given tax rate, the proceeds from taxing households would be lower. Thus, the resources available for financing public consumption would be reduced. However, this effect depends on the sign of $\frac{\partial l}{\partial B'}$ and corresponds to the term $\frac{TAF_i(l^*)}{(1+T)} \frac{\partial l}{\partial B'}$. Nevertheless, increasing $B'$ can only be optimal if it allows the government to increase $G$, in other words when $q + B' \frac{\partial q}{\partial B'} - \frac{TAF_i(l^*)}{(1+T)} \frac{\partial l}{\partial B'}$ is positive. In this case the extra resources are used to rise public spending, thus today’s utility goes up. Next period, the government would have to repay its debt, which reduces future government consumption. However, it only pays back when it is optimal, so households only end up enjoying less public goods in those states where the government does not default and honors its debt.

5. Quantitative Analysis

The model is solved numerically and the parameters are based on existing data and empirical work on emerging markets. Argentina is used as a benchmark as it represents the mayor case of sovereign default in recent history. However, many of the business cycles features observed in Argentina have been present in several emerging market
economies as well, Aguiar and Gopinath (2004). Given the availability of data on private and public consumption, the period of study starts in 1993.

5.1. Data

The data are seasonally adjusted quarterly real series obtained from the Ministry of Economy and Production (MECON) of Argentina. The interest rate spreads for Argentina are the Emerging Markets Bond Index (EMBI)\(^9\). Output, private consumption and public consumption are in logs and the trade balance is presented as a percentage of GDP. All series are filtered with the Hodrick-Prescott filter. Table 1 shows the business cycle statistics for Argentina.

With regard to the default rate, in the case of Argentina, Beim and Calomiris (2000) report three episodes of sovereign default in the last two hundred years, one during the nineteenth century and two during the twentieth century. In addition, Argentina defaulted one more time in 2001. Thus, it has defaulted four times in approximately two hundred years, implying a default rate of 2\(^%\)^{10}.

5.2. Calibration

The calibration involves choosing the functional forms and the parameter values. The parameters are chosen based on existing empirical work on emerging markets, if available. Otherwise they are set to mimic empirical regularities of emerging markets.

The per period utility function of households is specified as

---

\(^9\)The serie for the EMBI for Argentina starts in 1994.

\(^{10}\)If we assume that after a default episode countries stay in autarky three years on average and we exclude these years from the calculation of the default rate, then we would get a rate of 2.13%. 

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\[ U (C, G, 1 - l) = \left( C - \frac{l^{1+\Psi}}{1+\Psi} \right)^{1-\sigma} + G^{1-\sigma} \]  \hspace{1cm} (5.1)

The first part of the per period utility has the GHH (1988) specification, which has the property that the marginal rate of substitution between private consumption and labor is independent of consumption. Therefore, labor supply does not depend on the level of consumption and its elasticity is equal to \( \frac{1}{\Psi} \). In addition, public expenditures and private variables, i.e. \( C \) and \( l \), are separable.

Household produce goods using a linear technology, where labor is the only input.

\[ AF(l) = Al. \]

Given these preferences and technology, and combining the first order condition of household’s problem and her budget constraint, the following expression is obtained for the optimal labor supply of households

\[ l = \left[ \frac{A}{1+T} \right]^\frac{1}{\Psi}. \]

Household’s labor supply depends on the productivity factor, the tax rate and the parameters of the per period utility, \( \Psi \). A higher tax on consumption reduces the optimal level of labor, whereas a positive productivity shock increases it.

The productivity shock is assumed to follow an AR(1) process

\[ \ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t, \]

with \( E(\varepsilon) = 0 \) and \( E(\varepsilon^2) = \sigma^2 \).
There is an output loss in autarky, i.e., households produce less goods with the same amount of labor services. The productivity factor in autarky $h(A)$ is specified as $\phi E(A)$ with $\phi \in (0,1)$ following Arellano (2006). The parameters for the model are shown in table 2. Some of the parameter values that are used are standard for business cycles models in emerging markets. The parameter $\sigma$, the coefficient of relative risk aversion, is set equal to 2, a standard value (see Aguiar and Gopinath 2006). The parameter $\Psi$ is set to 0.455 following Mendoza (1991). This parameter determines the labor supply elasticity, which equals $\frac{1}{\Psi}$.

The discount factor $\beta$ is set at 0.95. One weakness of quantitative models of sovereign default is that they require a high level of impatience to generate default in equilibrium, so low values of the discount factor are employed to match the default rates. Aguiar and Gopinath (2006) and Yue (2006) use values of 0.80 and 0.74 respectively. In this paper a more reasonable value for the discount factor is used.

The parameter of the cost of default $\phi$, is calibrated to match the risk free debt limit in emerging markets. According to Reinhart, Rogoff and Savastano (2003) safe external debt to GDP thresholds for emerging economies appear to be as low as 15 to 20 percent of GDP. In the calibrated economy, the government does not have any incentive to default, and it faces risk free interest rates, for values of foreign debt up to 0.1093 units of goods. In terms of foreign debt-GDP ratio, this amount corresponds to 14.03% (20.70%) of GDP when the productivity factor takes its highest (lowest) value.

The parameter $\mu$ reflects the exogenous probability of reentering international capital markets after default and it is set equal to 0.1. This value implies that a defaulting country will return to financial markets in about 10 quarters after defaulting on its foreign debt. This is in line with the exclusion period observed in the data by Gelos
(2003), who calculated the average number of years that a country was excluded from international financial markets to be close to 3 years for countries that defaulted on their foreign debt during the period 1980 - 1999.

The risk free interest rate is set equal to 1%, which corresponds to the US quarterly interest rate and the parameters of the stochastic process for the productivity factor were set to match as closely as possible the observed cyclical properties of GDP in Argentina: the autocorrelation and standard deviation of the Argentinian output. In addition, the first order autoregressive process is approximated by a discrete first order Markov chain with 25 values using Hussey and Tauchen’s (1991) procedure.

Finally, given the values of the parameters, the preferences, which are specified in equation 5.1, allow the model to generate a public spending - private consumption ratio equal to 22%, which is close to the mean value for Argentina. During the period of study, from 1993 to 2005 this ratio fluctuated between 18% and 23%.

5.3. Results

The simulation results and the statistical properties of the model are presented in this section. Table 3 shows the business cycle moments of several macroeconomic variables for the simulated small open economy. Business cycles statistics are average values over 1000 simulations of 50 realizations each, drawn from a stationary distribution. The simulated series are logged and filtered.

The model economy is able to match several stylized facts in emerging markets: Incentives to default are higher for highly indebted countries, default risk and interest rates are countercyclical, public and private consumption are positively correlated with output, tax rates increase in recessions and fall in expansions and the trade balance
is negatively correlated with output, the country borrows more in good times at lower interest rates.

Figure 1 shows the default region for the calibrated economy, i.e., the combinations of productivity shocks and foreign debt levels for which default is the optimal decision. Given a shock, if default is optimal for a certain foreign assets level, it will be optimal for all lower levels of assets. In the same way, if repayment is optimal for a given amount of assets, it will be optimal for all higher levels of assets. Thus, for each realization of \( A \), default incentives are stronger for highly indebted governments, since given \( A \) the repayment of the outstanding debt is more costly the higher the amount of foreign debt. Analytically, this result is derived from the fact that the value of paying back and staying in good credit standing is strictly increasing in foreign assets, while the value of defaulting and going to autarky does not depend on the amount of foreign assets. Thus, if default is the optimal decision for some level of assets \( B \) given \( A \), then the value of staying in the contract is lower than the value of defaulting. A higher amount of foreign assets increases the value of the contract without affecting the value of default. Thus, for each value of \( A \) there exists a threshold \( B^* (A) \) for which the value of not default is equal to the value of default. For all levels of assets higher than \( B^* (A) \) the government optimally honors its debt. Figure 2 shows the value function of the government when it has the option to default or repay, given three values of the productivity shock. As we can see, a lower value of \( A \) increases the level of assets at which default is optimal, \( B^* (A_{\text{high}}) < B^* (A_{\text{low}}) \).

Figure 3 plots the discount bond price schedule as a function of assets for the highest and lowest values of the productivity shock, as well as the bond price for the middle value of the shock. As the figure shows, given a particular shock, bond prices are an
increasing function of foreign assets. For small levels of foreign debt, the government always pays back its debt, so it borrows from international markets at the world risk free interest rate. In this range of debt, bond price is simply the inverse of the gross risk free rate. However, as foreign debt goes up, at a certain debt level bond prices start to decrease. The higher the levels of foreign debt the lower the bond prices because the incentives to default are stronger for large indebted governments. At sufficiently large debt levels the government always defaults regardless of the productivity realization. At that point bond prices are zero since the probability of default is one. However, these values are not observed in the simulated economy, since a bond price equal to zero does not occur in equilibrium.

In addition, the bond price schedules also indicate that for all levels of debt, the government faces lower discount bond prices (higher interest rates) when the economy is hit by an adverse productivity shock. This result is due to the asset structure of the model, where there is only one asset, a one period non contingent bond. Given this market structure, defaulting on foreign debt is more attractive in bad times when output is low since the repayment of non contingent loans is more costly in recessions. Since productivity shocks are persistent, lending resources to the government in times of low output involves a higher default risk. Thus, risk neutral lenders are willing to lend resources to the government by charging a higher interest rate. The simulated sequence plotted in Figure 4 shows how the interest rate spread increases as output contracts in the economy.

The consumption tax and output are negatively correlated, implying that in bad times when the government has limited access to international credit markets it taxes more heavily on consumption in order to get resources to finance its expenditures. On
the other hand, in good times when foreign credits are cheaper, the government finances its expenditures mainly through external borrowing. Thus, the fact that the government is not able to commit to repay its debt induces a procyclical tax rate. Figure 5 shows the dynamics of GDP and tax rates in one simulation of the model economy.

In bad states of nature, the government relies more on taxes for the following reason. As mentioned before given the asset structure of the model, default is more tempting in recessions. Thus, if the government defaults then it is the case that an adverse shock has hit the economy. Therefore, in the middle of a recession, a highly indebted government loses access to credit markets and it has to finance its expenditures with taxes. However, if the productivity shock is low but the government is not highly indebted then it might find optimal to pay back and remain in international markets. Nevertheless, since the productivity shock is persistent a bad shock today implies a higher probability of having a bad shock tomorrow. Thus, the lower the productivity shock today the higher the default risk for a given amount of external borrowing. In this sense, even if the government does not default it faces higher interest rates in bad times, which implies that using external debt to finance public expenditures is a relatively less attractive option. Thus, the government finds it optimal to rely more heavily on taxes when financing public expenditures.

Figure 6 presents the tax policy function $T(B, A)$ as a function of $B$ for three values of the productivity shock. When the amount of foreign debt is low and the government always pays back its debt, the country faces the risk free interest rate and the optimal tax rate increases with the value of $A$. However, when foreign debt is high enough such that asset positions are carrying a positive risk premia, the optimal tax rate decreases when a positive productivity shock occurs. When the country is in autarky, the tax rate
falls since the government only has to collect tax revenues to finance its expenditures, without having to make payments to foreign lenders.

In the same way, public spending is procyclical since it is valued by households and markets are incomplete. The only asset in the economy is a one-period non-contingent bond. Since default is more attractive in bad times, it is the case that the bond price is low in recessions and high in expansions. The asset structure in the model and the counter-cyclical interest rates reduce the ability of the government to smooth consumption. Thus, both public and private consumption are highly correlated with output.

Finally, the trade balance is countercyclical in the calibrated economy, the country borrows more in good macroeconomic times at lower interest rates. The availability of external credit and the interest rate vary with the business cycle. Foreign lenders respond to an improvement in the domestic macroeconomic conditions by demanding a lower risk premium. Given the lower cost of foreign borrowing, the government views this as a good opportunity to finance expenditures cheaply and thus dissaves. Additionally, productivity shocks are persistent. If the economy is hit by a positive shock today then the probability of having good shocks in the future increases. The perspective of having a higher income tomorrow also induces the government to borrow more. Figure 7 shows the borrowing policy function $B'(B, A)$ conditional on not defaulting, as a function of $B$ for three values of the productivity shock. When the country is highly indebted, the government clearly borrows more in booms than in recessions.

6. Conclusions

We develop a stochastic dynamic equilibrium model of a small open economy that helps rationalize several stylized facts that are present in emerging market economies. Our
model features an endogenous fiscal policy in an incomplete credit markets framework where the country has the option to default on its sovereign debt. In our paper the government only defaults when the recession is sufficiently large and the government is already highly indebted. In that case the country is temporarily excluded from external markets and public expenditures are financed by taxation. Although defaulting is more tempting in bad times, an adverse shock does not always induce the government to default on its foreign debt. When the government faces a negative shock and does not default, it has the option to borrow from abroad, though at a high interest rate. Since the cost of external loans increases in recessions, financing public expenditures with foreign debt becomes a relatively less attractive option. Thus, complementing the previous literature that focused on the loss of market access, we find that the possibility of default and the associated risk premium play an important role in inducing a procyclical fiscal policy. In addition, since the non-contingent bond is not a good instrument for consumption smoothing purposes, the government is not able to smooth public spending, so public expenditures are highly procyclical. Therefore, the government optimally implements a procyclical fiscal policy. The model also generates countercyclical interest rate spreads and trade balance as well as procyclical private consumption.
7. Appendix

7.1. Algorithm

From the first order condition of the household's problem and her budget constraint, we can get the following expressions for the optimal labor supply:

\[ l^* = \left[ \frac{A}{1 + T} \right]^\frac{1}{\psi} \]

this expression is plugged in 3.2 and 3.7, to get the following expressions for private consumption and public expenditures:

\[ C^* = \frac{A l^*}{(1 + T)} \]
\[ G = TC^* + B - q \left( B', A \right) B' \]  \hspace{1cm} (7.1)

which are plugged in 5.1:

\[ U (C^*, G, 1 - l^*) = \frac{\left( C^* - \left( l^* \right)^{1+\psi} \right)^{1-\sigma}}{1 - \sigma} + \frac{G^{1-\sigma}}{1 - \sigma}. \]  \hspace{1cm} (7.2)

Assume an initial function for the price of the bond \( q_0(B', A) \) as well as initial values for \( V_0 \) and \( V^d \). To calculate the initial value of the bond, use the inverse of the risk free rate. For the initial values of the value functions, \( (V_0)_0 \) and \( (V^d)_0 \), start with 0 matrices, then the following algorithm is used:

1. Use \( q_0 \) to get \( G_0 \) from 7.1, then for every point \( (A, B, B') \) find the value of the tax rate, \( T_0 \), that maximizes 7.2.

2. Use \( q_0 \) and \( T_0 \) to express the per period utility 7.2 as a function of \( B, B' \), and \( A \), then use \( (V_0)_0 \) and \( (V^d)_0 \) and equations 3.3, 3.8 and 3.9 to get \( (V_0)_1 \), \( (V^d)_1 \), the
policy function, \( B'(B, A) \) and default function \( D(B, A) \) as well as the optimal tax rate associated with \( A, B \) and \( B'(B, A) \).

3. Given the default function \( D(B, A) \), and the repayment and default sets \( \Gamma(B) \) and \( \Gamma'(B) \), compute the probability of default \( \lambda(B', A) \) using 3.11.

4. Update the price of the bond using the following equation:

\[
q_1 = \frac{(1 - \lambda(B', A))}{1 + r_f}
\]

5. Use the updated price of the bond \( q_1 \) and the value functions \( (V_0)_1 \) and \( (V^d)_1 \) to repeat steps 1, 2, 3 and 4 until the following conditions are satisfied:

\[
\max \left\{ q_0 (B', A) - q_1 (B', A) \right\} < \epsilon
\]

\[
\max \left\{ (V_0 (B, A))_0 - (V_0 (B, A))_1 \right\} < \epsilon
\]

\[
\max \left\{ (V^d (A))_0 - (V^d (A))_1 \right\} < \epsilon
\]

where \( \epsilon \) is a small number
7.2. Derivation of First Order Conditions

The representative household maximizes 3.1 subject to 3.2, taking as given the government policies. From 3.2 we get an expression for private consumption, then we plug it into the household per period utility. The first order condition with respect to \( l \) is:

\[
U_c(C,G,1-l) \frac{\partial C}{\partial l} = U_l(C,G,1-l),
\]

\[
U_c(C,G,1-l) \frac{AF_l(l)}{(1+T)} = U_l(C,G,1-l).
\] (7.3)

Plugging the government budget constraint, equation 3.7, into equation 7.3 and solving for \( l \), we get the decision rule for labor supply \( l^* \) as a function of the government policies, \( T \) and \( B' \), as well as the state of the economy, \( A \) and \( B \). From the expression for private consumption we can also express the decision rule for private consumption \( C^* \) as a function of government policies and the state of the economy.

\[
C^*(A,B,T,B') = \frac{AF(l^*(A,B,T,B'))}{1+T}.
\] (7.4)

If we plug \( l^* \) and \( C^* \) into the government budget constraint, we get the following expression for public consumption

\[
G = AF(l^*(A,B,T,B')) + B - q(A,B') B' - C^*(A,B,T,B').
\] (7.5)

In order to solve the government problem, we plug 7.5 into equation 3.8. Therefore, the government sets \( T \) and \( B' \) to maximize household utility taking as given how the private sector reacts to the government policies.
From equation 3.8 we get the first order condition with respect to the tax rate, when the country has access to external borrowing

\[
U_c \left[ \frac{\partial C}{\partial T} + \frac{\partial C}{\partial l} \frac{\partial l}{\partial T} \right] + U_g \left[ \frac{\partial G}{\partial C} \frac{\partial C}{\partial l} + \frac{\partial G}{\partial l} \frac{\partial l}{\partial T} \right] + U_l \frac{\partial l}{\partial T} = 0 \quad (7.6)
\]

Rearranging terms:

\[
\left[ U_c \frac{\partial C}{\partial l} - U_l \right] \frac{\partial l}{\partial T} + U_g \left[ \frac{\partial G}{\partial C} \frac{\partial C}{\partial l} + \frac{\partial G}{\partial l} \frac{\partial l}{\partial T} \right] + \left[ U_c + U_g \frac{\partial G}{\partial l} \right] \frac{\partial C}{\partial T} = 0
\]

The first term corresponds to the household first order condition, thus it is equal to zero.

\[
U_g \left[ \frac{\partial G}{\partial C} \frac{\partial C}{\partial l} + \frac{\partial G}{\partial l} \frac{\partial l}{\partial T} \right] + \left[ U_c + U_g \frac{\partial G}{\partial l} \right] \frac{\partial C}{\partial T} = 0
\]

Taking the derivative of 7.5 with respect to \(C\) and \(l\), as well as the derivative of 7.4 with respect to \(l\), then plugging them into the government first order condition for the tax rate, we get the following expression.

\[
U_c (C, G, 1 - l) \frac{AF (l^*)}{(1 + T)^2} = U_g (C, G, 1 - l) \left[ \frac{AF (l^*)}{(1 + T)^2} + \frac{TAF_l (l^*)}{(1 + T)} \frac{\partial l}{\partial T} \right].
\]

In order to get the Euler equation of the government, we derive equation 3.8 with respect to \(B'\).

\[
0 = U_c \frac{\partial C}{\partial l} \frac{\partial l}{\partial B'} - U_l \frac{\partial l}{\partial B'} + U_g \left[ \frac{\partial G}{\partial C} \frac{\partial C}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial q} \frac{\partial q}{\partial B'} + \frac{\partial G}{\partial B'} \right] + \ldots
\]

\[
... + \beta \sum_{A' \in \Gamma (B)} \frac{\partial V_c}{\partial B'} Q(A'/A)
\]
The first term of the above expression corresponds to the first order condition from the household problem and it is equal to zero. Hence

\[ U_g \left[ \frac{\partial G}{\partial C} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial q} \frac{\partial q}{\partial B'} + \frac{\partial G}{\partial B} \right] + \beta \sum_{A' \in \Gamma(B)} \frac{\partial V^c}{\partial B'} Q(A'/A) = 0 \]  \hspace{1cm} (7.7)

Taking the derivative of 7.5 with respect to \( C, l, q \) and \( B' \) as well as the derivative of 7.4 with respect to \( l \), then plugging them into the government first order condition for external borrowing, we get the following expression.

\[ U_g \left[ \frac{T A F_l (l^*)}{1 + \tau} \frac{\partial l}{\partial B'} - B' \frac{\partial q}{\partial B'} - q \right] + \beta \sum_{A' \in \Gamma(B)} \frac{\partial V^c}{\partial B'} Q(A'/A) = 0 \]  \hspace{1cm} (7.8)

To obtain an expression for \( \frac{\partial V^c}{\partial B} \), we differentiate equation 3.8

\[ \frac{\partial V^c}{\partial B} = \frac{\partial C}{\partial l} \left[ \frac{\partial l}{\partial B'} + \frac{\partial l}{\partial T} \frac{\partial T}{\partial B'} + \frac{\partial C}{\partial B'} \right] + \frac{\partial l}{\partial T} \frac{\partial C}{\partial B'} - U_l \left[ \frac{\partial l}{\partial B'} + \frac{\partial l}{\partial T} \frac{\partial T}{\partial B'} \right] + ... \]

\[ + \frac{\partial G}{\partial C} \left[ \frac{\partial C}{\partial B'} + \frac{\partial C}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial C}{\partial T} \frac{\partial T}{\partial B'} \right] + \frac{\partial G}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial T} \frac{\partial T}{\partial B'} + \beta \sum_{A' \in \Gamma(B)} \frac{\partial V^c}{\partial B'} Q(A'/A) \frac{\partial B'}{\partial B} \]

The terms multiplying \( \frac{\partial T}{\partial B} \) correspond to equation 7.6, which is equal to zero

\[ \frac{\partial V^c}{\partial B} = \left[ U_c \frac{\partial C}{\partial l} - U_l \right] \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial C} \left[ \frac{\partial C}{\partial B'} + \frac{\partial C}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial C}{\partial T} \frac{\partial T}{\partial B'} \right] + ... \]

\[ + \frac{\partial G}{\partial l} \frac{\partial l}{\partial B'} + \frac{\partial G}{\partial q} \frac{\partial q}{\partial B'} + \frac{\partial G}{\partial B'} \frac{\partial B'}{\partial B} + \beta \sum_{A' \in \Gamma(B)} \frac{\partial V^c}{\partial B'} Q(A'/A) \frac{\partial B'}{\partial B} \]

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The first term is equal to the first order condition from the household problem, which is equal to zero and the rest of the terms multiplying $\frac{\partial B'}{\partial B}$ are equal to equation 7.7 which is also equal to zero. Finally the derivative of 7.5 with respect $B$ is equal to 1. Thus, we get the following expression

$$\frac{\partial V^c}{\partial B} = U_g$$

Updating the above equation and plug it into equation 7.8 we get the Euler equation for the government

$$U_g(C, G, 1 - l) \left[ B' \frac{\partial q}{\partial B'} + q - \frac{TAF_l(l^*)}{(1 + T)} \frac{\partial l}{\partial B'} \right] = \beta \sum_{A' \in \Gamma(B)} U_g(C', G', 1 - l') Q(A'/A)$$
7.3. Tables and Figures

Table 1. Argentinian Data (1993-2005)

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$\sigma (GDP)$</td>
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<td>$\sigma (\text{Spread})$</td>
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<td>$\rho (GDP, G)$</td>
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<td>$\rho (GDP, TB/GDP)$</td>
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<td>Default Rate (%)</td>
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Table 2. Parameter Values

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</tr>
<tr>
<td>$\rho(GDP, \text{Public Consumption})$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho(GDP, \text{Tax Rate})$</td>
<td>-0.33</td>
</tr>
<tr>
<td>$\rho(GDP, TB/GDP)$</td>
<td>-0.34</td>
</tr>
<tr>
<td>Default Rate (%)</td>
<td>0.26</td>
</tr>
<tr>
<td>Max Tax Rate (%)</td>
<td>24.18</td>
</tr>
<tr>
<td>Max Spread (%)</td>
<td>1.15</td>
</tr>
</tbody>
</table>
Figure 1 Default Region

Figure 2 Value Function Vo(B,A)
Figure 3 Bond Price

Figure 4 Dynamics of GDP and Spread
Figure 5 Dynamics of GDP and Tax Rate

Figure 6 Tax Rate Policy T(B,A)
Figure 7 Borrowing Function $B'(B,A)$
References


