The Financial Accelerator from a Business Cycle Accounting Perspective

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Abstract
In a recent paper, Gertler, Gilchrist and Natalucci (2006) report that the financial accelerator mechanism may account for about half of the fall in output and investment observed during the Korean crisis of 1997-1998. Using the business cycle accounting method of Chari, Kehoe and McGrattan (2006a), this paper finds that such a result is very sensitive to the value of Tobin’s q elasticity. The implication is that the adjustment cost function may be crucial in terms of the relative importance of distortions for explaining business cycle fluctuations.

Keywords: Business cycle accounting, Financial accelerator, Korean crisis.
JEL Classification: E1, E32, F4.

Resumen
En un artículo reciente, Gertler, Gilchrist y Natalucci (2006) encuentran que el mecanismo del acelerador financiero es capaz de explicar alrededor de la mitad de la caída en producción e inversión durante la crisis de Corea de 1997-1998. Al aplicar la metodología de contabilidad de ciclos económicos de Chari, Kehoe y McGrattan (2006a), el presente documento encuentra que dicho resultado es muy sensible al valor de la elasticidad de q de Tobin. Esto implica que la función de costos de ajuste puede ser crucial para entender la importancia relativa de las distintas distorsiones capaces de explicar los ciclos económicos.

Palabras Clave: Contabilidad de ciclos económicos, Acelerador financiero, Crisis de Corea.

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1 Introduction

In recent years, there has been an increasing interest in trying to understand the transmission mechanism of financial crisis to the real economy in dynamic macroeconomic models. This research has been motivated largely in part by the financial crises registered in developing countries over the past twenty five years. One example of this type of crisis is given by Korea in the late nineties. The crisis roughly started in October 1997 when the country’s sovereign risk status was downgraded at a time when Korean off-shore banks were exposed to large dollar-denominated foreign loans. Given the massive capital flight, the central bank responded by raising the overnight rate over a thousand basis points to defend the (fixed) exchange rate. The real effects of the crisis were evident soon thereafter. For example, seasonally adjusted real GDP fell about 9 percent in the first quarter of 1998, whereas investment and hours worked fell about 12 and 10 percent, respectively. Although government expenditures and exports remained relatively stable with respect to the previous quarter, imports decreased about 21 percent in the first quarter of 1998. Thus, the fall of real variables in Korea was far from trivial.\(^1\)

In accounting for the Korean crisis of 1997-1998, Gertler, Gilchrist and Natalucci (2006, henceforth GGN) construct an otherwise standard small open economy model modified to allow for financial frictions of the type originally developed by Bernanke and Gertler (1989), and explored by Carlstrom and Fuerst (1997) and Bernanke et al. (1999).\(^2\) After calibrating the model to replicate key characteristics of the Korean economy, GGN find that the financial accelerator mechanism of Bernanke and Gertler (1989) accounts for nearly half of the decline in economic activity observed in Korea during the 1997-1998 crisis. The reason is that the financial accelerator mechanism helps to magnify the effect of shocks to the economy. In these models, firms’ cost of capital is directly linked to their own financial position in the sense that less leveraged firms have to pay a lower premium to finance their capital acquisition. In the event of an adverse shock that damages the balance sheet of a firm, the cost of capital increases which leads to a contraction in investment and output, thus magnifying the original effect of the shock.

\(^1\) For a detailed description of the Korean crisis, see among others Shin and Hahm (1998), Koo and Kiser (2001) and the references therein.

\(^2\) This type of financial friction can also be found in Céspedes et al. (2004), Cook (2004), and Tovar (2005, 2006), among others.
Here, the financial accelerator mechanism is studied from a business cycle accounting perspective as developed by Chari, Kehoe and McGrattan (2006a). The general idea of this method is to show that a large class of dynamic equilibrium models are equivalent to a prototype model with time-varying wedges. These wedges represent deviations between inputs and outputs as well as distortions in the prototype model’s first-order conditions. Following Chari et al. (2006a), they are labeled efficiency, labor, investment, and government consumption wedges. Conveniently, these four wedges may be estimated from data and then fed back into the prototype model in order to assess how much of the movements in variables such as output, labor and investment may be attributed to each wedge, either separately or in combinations. By construction, the wedges in the prototype economy account for all of the observed movements in the data.

The goal of this paper is to evaluate the robustness of the results reported by GGN using the business cycle accounting method. The benchmark prototype model of Chari et al. (2006a) is slightly modified to include adjustment costs, following the suggestions of Christiano and Davis (2006). First it is shown that the finance premium that captures the financial accelerator mechanism in the model of GGN may be represented in terms of an investment wedge in the prototype economy.\footnote{A similar proof may be found in Christiano and Davis (2006) in the context of a simpler model with flexible prices. Aoki et al. (2004) consider an economy with financial frictions in the spirit of Bernanke et al. (1999) applied to the credit market for households rather than firms. In such a model, the finance premium may also be represented in terms of an investment wedge in the prototype economy.} From a business cycle accounting perspective, the investment wedge captures the distortions in the Euler equation of agents otherwise operating in competitive markets. Next, simulation exercises are performed to evaluate the importance of the investment wedge, separately and in combinations, to account for movements in output, investment and labor observed in Korean data.

Remarkably, the business cycle accounting exercise is able to replicate the findings of GGN in the sense that the financial accelerator mechanism alone may explain about a half of the fall in output and investment during the Korean crisis if the relatively small Tobin’s $q$ elasticity value of 0.5 used by GGN is assumed. Such an elasticity value may seem small as Elekdag et al. (2006) and Lubik and Teo (2005) suggest values for this parameter between 1.8 and 3.2 using Bayesian techniques in a fully specified dynamic equilibrium model. In fact, from a business cycle accounting perspective the result reported by GGN is very sensitive to the parametrization of the adjustment cost. If larger values for Tobin’s $q$ elasticity are used
instead as suggested by Elekdag et al. (2006) and Lubik and Teo (2005), the investment wedge now explains a lower fraction of macroeconomic fluctuations. In the extreme case where the Tobin’s q elasticity goes to infinity, the investment wedge alone would be consistent with a rise in output right after the crisis, a result completely at odds with the data. This sensitivity arises because the investment wedge is specified in terms of the adjustment cost function: varying the Tobin’s q elasticity rescales the investment wedge, and such a rescaling alters the response of macroeconomic variables to this particular wedge. The sensitivity of the investment wedge to Tobin’s q elasticity is also found under alternative specifications of the prototype economy, such as the introduction of variable capital utilization and measurement errors. The major implication of this finding is that the specification of the adjustment cost function may play a key role in dynamic equilibrium models as it may affect the relative importance of a particular wedge for explaining business cycle fluctuations.

The rest of the paper is divided in three sections. Section two describes the three models used in the business cycle accounting exercise. First the benchmark prototype economy of Chari et al. (2006a) with adjustment costs is presented along with a characterization of the four wedges mentioned earlier. Next, the financial accelerator model of GGN with nominal price rigidities is discussed in detail, and it is shown that the financial accelerator mechanism may be captured by an investment wedge in the associated prototype economy. Section three discusses the estimation method for the wedges and presents a series of simulations under alternative Tobin’s q elasticity values and combinations of wedges. Section four concludes.

2 The Models

The business cycle accounting method of Chari et al. (2006a) has two basic components: an accounting procedure and an equivalence result. The method usually requires three models to recover the wedges from the data and to give them an economic interpretation. The first model (labeled the “benchmark prototype economy”) is used exclusively for the accounting procedure. It considers a roughly standard neoclassical growth model with four stochastic variables or wedges: efficiency, labor, investment, and government consumption wedges. These time-varying wedges distort the equilibrium decisions of agents operating in otherwise competitive markets. They are first estimated from both the data and the equilibrium conditions of the benchmark prototype economy, and then fed back into the model to quantitatively account for the contribution of wedges to business cycle fluctuations, either
separately or in combinations. For example, the importance of the investment wedge to explain movements in macroeconomic variables may be assessed by cancelling the contribution of the other three wedges in the model. By construction, the four wedges fully account for the observed movements in macroeconomic variables.

The remaining two models are used for the equivalence result. This result is useful as it yields an economic interpretation of the wedges. The first of these models is labeled the “detailed model” in the sense that, compared to a standard neoclassical growth model, it includes as many distortions as necessary to capture some characteristics of the data. The second model is referred to as the “associated prototype economy”, which is a version of the benchmark prototype economy constructed in such a way that it has the same aggregate allocations as the detailed model (the equivalence result). Thus the distortions of the detailed model may conveniently be expressed in terms of wedges in the associated prototype economy. In this sense, the associated prototype economy has the sole purpose of giving an economic interpretation to the wedges estimated in the accounting procedure. This is important since the wedges do not have a unique economic interpretation as there is a large class of models that can be represented in terms of such wedges. For this reason, the business cycle accounting method does not uniquely determine the model most promising to study business cycle fluctuations. It does, however, provide a useful guide for researchers about the distortions that are key to explain macroeconomic fluctuations.

To see how the method works, this section presents the three models referred to above. The benchmark prototype economy is extended to include adjustment costs for investment, as Christiano and Davis (2006) find that the accounting exercise may be sensitive to the value of Tobin’s q elasticity. The detailed model is simply the small open economy with a financial accelerator mechanism of Gertler et al. (2006). Finally, the associated prototype economy is constructed so that it is equivalent to the model of Gertler et al. (2006). In such a case, the equivalence result is useful to show that the financial accelerator mechanism may be captured by the investment wedge in the associated prototype economy. Given this result, the benchmark prototype economy may then be used in order to quantitatively assess the contribution of the financial accelerator mechanism to business cycle fluctuations.
2.1 The Benchmark Prototype Economy

A standard neoclassical growth model with adjustment costs is considered for the business cycle accounting exercise. As in Chari et al. (2006a), four stochastic variables are included: the efficiency wedge $A_t$, the labor wedge $1 - \tau_{n,t}$, the investment wedge $1/(1 + \tilde{\tau}_{x,t})$, and the government consumption wedge $g_t$.

Consumers in this economy choose per capita consumption $c_t$ and per capita labor $l_t$ in order to maximize lifetime expected utility, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t$$

subject to the budget constraint

$$c_t + (1 + \tau_{x,t})x_t = (1 - \tau_{n,t})w_tl_t + r_tk_t + T_t$$

and the law of motion for capital

$$(1 + n)k_{t+1} = (1 - \delta)k_t + x_t - \varphi(x_t/k_t)k_t, (1)$$

where $N_t$ is the period $t$ population growing at the rate $1 + n$, $x_t$ is investment, $w_t$ the wage rate, $r_t$ the rental rate of capital, $k_t$ the per capita capital stock, $T_t$ the per capita lump-sum transfers, $\delta$ the depreciation rate of capital with $0 < \delta < 1$, $\beta$ a discount factor satisfying $0 < \beta < 1$, and $\tau_{x,t}$ and $\tau_{n,t}$ the tax rates on investment and labor, respectively. The function $\varphi(x_t/k_t)$ represents adjustment costs for investment with properties $\varphi' > 0$ and $\varphi'' \geq 0$.

Technology in this economy is given by a neoclassical production function of the form

$$F(k_t, (1 + \gamma)^tl_t)$$

where $(1 + \gamma)^t$ is the exogenous growth rate of labor-augmenting technical progress. In per capita terms, output $y_t$ is determined by

$$y_t = A_t F(k_t, (1 + \gamma)^tl_t). (2)$$

As is usual in a perfectly competitive environment, prices of each factor of production are equal to their corresponding marginal productivities, i.e., $w_t = F_{n,t}$ and $r_t = F_{k,t}$.

Finally, there is a government with an exogenous level of per capita expenditures $g_t$. The corresponding resource constraint in this economy is given by

$$c_t + x_t + g_t = y_t. (3)$$
Standard first-order conditions of the household problem yield

\[-\frac{U_{t,t}}{U_{c,t}} = (1 - \tau_{n,t}) A_t (1 + \gamma) t F_{n,t}, \tag{4}\]

and

\[(1 + \tilde{\tau}_{x,t}) U_{c,t} = \beta E_t U_{c,t+1} \left[ A_{t+1} F_{k,t+1} + (1 + \tilde{\tau}_{x,t+1}) \Gamma_{t+1} \right], \tag{5}\]

where \(U_{j,t}\) denotes the derivative of \(U_t\) with respect to \(j\), \(1 + \tilde{\tau}_{x,t} \equiv \frac{1 + \tau_{x,t}}{1 - \varphi'(x_t/k_t)}\), and \(\Gamma_{t+1} \equiv \left(1 - \delta - \varphi \left(\frac{x_{t+1}}{k_{t+1}}\right) + \varphi' \left(\frac{x_{t+1}}{k_{t+1}}\right) \left(\frac{x_{t+1}}{k_{t+1}}\right)\right)\). Equation (4) is the marginal rate of substitution between leisure and consumption, which is equal to the after-tax marginal product of labor. Expression (5) is the familiar Euler equation, where intertemporal consumption is a function of the investment tax rate \(\tilde{\tau}_{x,t}\).

For this benchmark prototype economy, the efficiency wedge \(A_t\) in (2) resembles the productivity parameter. In a similar fashion, the terms \(1 - \tau_{n,t}\) and \(1/(1 + \tilde{\tau}_{x,t})\) introduce a wedge in expressions (4) and (5) with respect to an otherwise standard neoclassical model with no distortions. These wedges resemble (but are not necessarily equal to) tax rates on labor income and investment. Finally, the government consumption wedge \(g_t\) is included in (3).

Notably, Chari et al. (2006a) show that a large class of macroeconomic models may be mapped into the benchmark prototype economy described above. For example, an economy with sticky wages and monetary shocks is equivalent to a prototype model with labor wedges. Alternatively, a model with constant technology and input-financing frictions is equivalent to a growth model with efficiency wedges. An open economy model with international borrowing and lending is equivalent to a prototype, closed economy model with a government consumption wedge, and so on. In this paper, an equivalence result for the detailed model of GGN is presented below.

In the benchmark prototype economy, each wedge in isolation captures the overall distortion or deviation between inputs and outputs to an equilibrium condition of the model. For example, distortions in the consumer’s intertemporal Euler equation may arise from liquidity constraints on consumers, whereas the firm’s intertemporal Euler equation may be affected

\[\text{4 This intertemporal wedge may alternatively be defined in terms of a tax on capital income} \ \tau_{k,t}. \ \text{Chari et al. (2006b) find that the accounting procedure is not sensitive to this alternative specification of the intertemporal wedge.}\]
by financial frictions on firms. Thus the investment wedge in (5) conveniently summarizes these two distortions. Therefore, if two or more distortions affect a particular equilibrium condition of the model, this method cannot identify each of them separately.

2.2 The detailed model of GGN

The model of GGN (2006) is a relatively standard small open economy model with money and nominal price rigidities. The major departure is the introduction of a financial accelerator mechanism in the spirit of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997). The idea of the financial accelerator is to create a link between the balance sheet of the borrower and the terms of credit, which is crucial for the demand of capital. As illustrated by Gertler et al. (2006), this mechanism can magnify the effects of shocks to the economy.

In the model, households make decisions about tradable consumption goods, labor supply, money balances, and domestic and foreign bonds. Tradable goods are produced both at home (H) and abroad (F). These goods are imperfect substitutes. In addition, there are three types of producers: (i) entrepreneurs; (ii) capital producers; and (iii) retailers. Entrepreneurs are the owners of capital. To produce wholesale goods, entrepreneurs need to borrow from households to finance the acquisition of capital required in the production process. Here, the financial accelerator mechanism makes the demand of capital dependent on the financial position of the firm. Capital producers have the role of building new capital to satisfy the demand of entrepreneurs. Finally, retailers buy wholesale goods from entrepreneurs and modify them slightly to produce final goods. A monopolistically competitive environment is assumed for the retail sector so that nominal prices may be set on a staggered basis. For the sake of brevity, the problem of households, entrepreneurs and capital producers is described below. Both the foreign sector and the retailer’s problem are presented in Appendix A.

2.2.1 Households

An infinitively-lived, representative household has lifetime utility of the form

\[ E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, H_t, \frac{M_t}{P_t} \right) \]

with
\[ U \left( C_t, H_t, \frac{M_t}{P_t} \right) = \frac{\left[ \left( C_t \right)^{1-\varsigma} (1 - H_t)^\varsigma \right]^{1-\sigma}}{1 - \sigma} + \xi \log \left( \frac{M_t}{P_t} \right), \]  

(7)

and \( \sigma \geq 0, \varsigma \in (0, 1), \xi > 0 \). Here, \( H_t \) and \( M_t/P_t \) denote labor supply and real money balances, respectively. Aggregate consumption \( C_t \) is a composite of both home consumption, \( C^H_t \), and foreign consumption, \( C^F_t \), according to the following CES specification:

\[ C_t = \left[ (\gamma)^{\frac{1}{\rho}} \left( C^H_t \right)^{\frac{\rho-1}{\rho}} + (1 - \gamma)^{\frac{1}{\rho}} \left( C^F_t \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \]  

(8)

The corresponding consumer price index (CPI), \( P_t \), is given by

\[ P_t = \left[ (\gamma) \left( P^H_t \right)^{1-\rho} + (1 - \gamma) \left( P^F_t \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \]  

(9)

The household derives income from labor, real dividend payments \( \Pi_t \) from ownership of retail firms, real money balances carried out from the previous period \( M_{t-1}/P_t \), and interest from home and foreign bonds (\( B_{t+1} \) and \( B^{*}_{t+1} \), respectively) net of real lump-sum taxes \( T_t \). Income is allocated to consumption and holdings of money, home and foreign bonds. Accordingly, the household budget constraint may be written as

\[ C_t = \frac{W_t H_t}{P_t} + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{B_{t+1} - (1 + i_{t-1}) B_t}{P_t} - \frac{S_t B^{*}_{t+1} - S_t \Psi_t (1 + i^*_{t-1}) B^{*}_t}{P_t}, \]  

(10)

where \( W_t \) denotes the nominal wage, \( (1 + i_t) \) and \( (1 + i^*_t) \) are the domestic and (exogenous) foreign gross nominal interest rate, respectively, \( S_t \) is the nominal exchange rate, and \( \Psi_t \) is the borrowing premium paid by domestic residents to foreign lenders. The premium \( \Psi_t \) is a function of net foreign indebtedness, \( NF_t \), and a random shock \( \varrho_t \) so that \( \Psi_t = f(BNF_t) \varrho_t \), where \( f'(\cdot) > 0 \). As is well known (cf. Schmitt-Grohe and Uribe (2003)), the country borrowing premium is introduced to avoid non-stationary net foreign indebtedness. Here, the borrowing premium function is a simple way to model an unexpected capital output flow, represented by an increase in the random variable \( \varrho_t \) which in turn directly affects \( \Psi_t \).

The household thus chooses aggregate consumption, labor, money balances, and domestic and foreign bonds to maximize (6) subject to (7) and (10).
2.2.2 Firms

Entrepreneurs and the Financial Accelerator. Entrepreneurs produce domestic output, $Y_t$, at the wholesale level using labor, $L_t$, and capital services, $u_tK_t$, according to the following process:

$$Y_t = \omega_tA_t(u_tK_t)^\alpha L_t^{1-\alpha},$$

where $u_t$ is the capital utilization rate and $K_t$ is the capital stock acquired in the previous period. $^5$ Here, $A_t$ is a common productivity factor and $\omega_t$ is an idiosyncratic, i.i.d. random variable, distributed continuously with $E\{\omega_t\} = 1$. Labor is assumed to be a composite of household and managerial labor ($H_t$ and $H^e_t$, respectively) according to $L_t = H_t^\Omega H^e_t^{(1-\Omega)}$. For convenience, $H^e_t$ is normalized to unity. Labor is hired at the competitive wage rate $W_t$ whereas entrepreneurs receive a small wage $W^e_t$ in compensation for their work.

Entrepreneur’s gross output, $GY_t$, is given by the sum of output revenues, $\frac{P_{W,t}}{P_Y}Y_t$, and the market value of the remaining capital stock, $Q_t\omega_tK_t$, net of the repairing cost of capital, $\frac{P_{I,t}}{P_t}\delta_t\omega_tK_t$. Here, $P_{W,t}$ denotes the nominal price of wholesale production, $Q_t$ the real market price of capital in terms of the consumption index (8), $P_{I,t}$ the nominal replacement price of capital, and $\delta_t$ the depreciation rate of capital. Hence,

$$GY_t \equiv \frac{P_{W,t}}{P_t}Y_t + \left(Q_t - \frac{P_{I,t}}{P_t}\delta_t\right)\omega_tK_t.$$ 

Finally, the utilization decision $u_t$ is endogeneized by assuming that the capital depreciation rate $\delta_t$ is increasing in $u_t$ according to the following convex function:

$$\delta(u_t) = \delta + \frac{b}{1+\xi}(u_t)^{1+\xi}, \ \delta, b, \xi > 0.$$

The entrepreneur’s problem is thus to choose labor and the capital utilization rate to maximize profits, conditioning on $K_t$, $A_t$ and $\omega_t$. In addition, they consume an amount of $C_t^uH$ units of the tradable domestic good.

Entrepreneurs also face a capital acquisition problem. In particular, they need to acquire capital at the end of period $t$ for the production process in $t+1$. The acquisition of capital

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$^5$ Entrepreneur-specific indices are omitted for notational simplicity.
is financed partly with entrepreneur’s own net worth at the end of period \( t \), \( N_{t+1} \), and partly by issuing nominal bonds, \( B_{t+1} \). Hence, capital financing \( Q_tK_{t+1} \) is given by

\[
Q_tK_{t+1} = N_{t+1} + \frac{B_{t+1}}{P_t}.
\]

In order to avoid that entrepreneurs accumulate enough funds to self-finance their capital acquisition, it is assumed that they have a finite expected horizon in the sense that entrepreneurs survive until next period with probability \( \phi \). It is also assumed that new equity issues are too expensive so that all marginal finance is obtained through debt only. In the model, debt is denominated in units of domestic currency.

The entrepreneur’s demand for capital depends on both its expected marginal return and its expected marginal financing cost. It may be shown that the expected marginal return of capital may be expressed as

\[
E_t \{ 1 + r^k_{t+1} \} = \frac{E_t \left\{ \frac{P W_{t+1}}{P_{t+1}} F_{k,t+1} - \frac{P_{t+1}}{P_{t+1}} \delta (u_{t+1}) + Q_{t+1} \right\}}{Q_t},
\]

where \( F_{k,t+1} \) is the marginal product of capital at \( t+1 \). On the other hand, the marginal cost of financing depends on firm’s own financial condition. Following Bernanke et al. (1999), an agency problem between lenders and borrowers with costly state verification is assumed. This agency problem arises because the idiosyncratic shock \( \omega_t \) is private information for the entrepreneur. Therefore, the lender can only observe the project’s gross output after paying an auditing cost, which is a fixed proportion \( \mu_h \) of the project’s ex-post gross return \( (1 + r^k_{t+1})Q_tK_{t+1} \). The financial contract guarantees that the entrepreneur does not have an incentive to misrepresent his earnings and that the expected agency cost is minimized. Under this contract, the lender charges the borrower a premium \( \chi_t(\cdot) \) to cover the expected auditing cost. It may be shown that the finance premium varies inversely with the entrepreneur’s net worth but directly with the leverage ratio, \( \frac{B_{t+1}/P_t}{N_{t+1}} \), according to

\[
\chi_t(\cdot) = \chi \left( \frac{B_{t+1}/P_t}{N_{t+1}} \right), \quad \chi'(\cdot) > 0, \quad \chi(0) = 0, \quad \chi(\infty) = \infty.
\]

\[6\] Gertler et al. (2006) consider debt denominated in domestic currency as it accounted for about 75 percent of total commercial banks’ liabilities at the beginning of 1998 in Korea. Cook (2004) presents a small open economy model similar to GGN with foreign-denominated debt. The finding described below where the financial accelerator may be represented by an investment wedge in the associated prototype economy does not depend on whether debt is domestic or foreign denominated.
In the model of GGN, the entrepreneur’s marginal costs of funds is given by the gross premium of external funds times the gross real opportunity cost of funds should capital market frictions be absent. Thus, the demand of capital satisfies

\[ E_t \{ 1 + r^k_{t+1} \} = [1 + \chi_t(\cdot)] E_t \left\{ (1 + i_t) \left( \frac{P_t}{P_{t+1}} \right) \right\}. \]  

(15)

Expression (15) captures the idea of the financial accelerator in the sense that the entrepreneur’s financial position is linked to the marginal cost of funds and thus to the demand for capital. In particular, a higher leverage ratio translates into a higher finance premium for the entrepreneur, thereby decreasing his demand for capital.

Finally, entrepreneur’s net worth \( N_{t+1} \) is given by

\[ N_{t+1} = \phi V_t + W^e_t / P_t, \]

where \( V_t \) is the value of capital net of borrowing costs carried over from the previous period.

**Capital Producers.** The role of capital producers in the model is to repair depreciated capital and to construct new capital goods in a competitive environment. Both of these activities are made after production of output at time \( t \) takes place, and use as input an investment good composed of domestic and foreign investment goods:

\[ I_t = \left[ (\gamma_i) \frac{1}{\rho} (I_t^H)^{\frac{\rho-1}{\rho}} + (1 - \gamma_i) \frac{1}{\rho} (I_t^F)^{\frac{\rho-1}{\rho}} \right] \frac{\rho}{\rho_i-1}. \]  

(16)

The corresponding investment price index, \( P_{t,t} \), is given by

\[ P_{t,t} = \left[ (\gamma_i) (P_t^H)^{1-\rho_i} + (1 - \gamma_i) (P_t^F)^{1-\rho_i} \right] \frac{1}{1-\rho_i}. \]  

(17)

For this particular model, the construction of new capital goods is subject to adjustment costs of the form \( \Phi \left( \frac{I_t^n}{K_t} \right) \), where \( I_t^n \) is net investment, i.e., \( I_t^n = I_t - \delta(u_t)K_t \). The function \( \Phi(\cdot) \) is increasing and concave. Given the constant returns to scale technology \( \Phi \left( \frac{I_t^n}{K_t} \right) K_t \), the aggregate capital accumulation equation is

\[ K_{t+1} = K_t + \Phi \left( \frac{I_t^n}{K_t} \right) K_t. \]  

(18)

In this framework, capital producers choose inputs \( I_t^n \) and \( K_t \) to maximize expected profits from the construction of new investment goods. New capital goods are sold at a price
$Q_t$ and capital is leased from entrepreneurs at the rate $r^l_t$. Finally, repair of old capital goods require $\delta(u_t)K_t$ units of the investment good. The repair cost is given by $\frac{P^H_t}{P_t} \delta_t K_t$, which is paid by the entrepreneurs as they are the owners of the capital stock (see equation 12).

**Government and Monetary Policy.** At each period $t$, exogenous government expenditures $G^H_t$ are financed by lump-sum taxes and seigniorage according to

$$\frac{P^H_t}{P_t} G^H_t = \frac{M_t - M_{t-1}}{P_t} + T_t. \quad (19)$$

To close the model, Gertler et al. (2006) specify two alternative monetary policy rules depending on whether the exchange rate regime is either fixed or flexible. Under a fixed exchange rate regime, the monetary authority sets the nominal interest rate so that the uncovered interest parity condition is satisfied. Under a flexible exchange rate regime, the central bank adopts a Taylor-type rule whereby the nominal interest rate is a function of deviations of inflation and domestic output from their corresponding target values. In each case, the only role for money holdings in the model is to pin down the nominal money stock.

**Resource Constraint.** Finally, the resource constraint for this economy is given by

$$\frac{P^H_t}{P_t} \left( C^H_t + C^eH_t + C^H_t + I^H_t + G^H_t \right) = \frac{P^H_t}{P_t} Y^H_t + \frac{P^F_t}{P_t} \left( C^F_t + I^F_t \right). \quad (20)$$

where $C^H_t$ is the foreign demand for the home tradable good and $Y^H_t$ is the final domestic good (both described in Appendix A).

### 2.3 The Associated Prototype Economy

Following Chari et al. (2006a), the goal now is to construct a prototype economy with relevant wedges so that it has the same aggregate allocations as those obtained under the model of Gertler et al. (2006) described earlier. As it will become clear shortly, the prototype economy requires efficiency, labor, investment and government consumption wedges. In particular, it will be shown that the finance premium (14) which is crucial for the financial accelerator mechanism in the model of Gertler et al. (2006) can be mapped into a investment wedge in the associated prototype economy.
In this economy, households maximize (6) subject to (7) and their budget constraint now given by:

\[ C_t + (1 + \tau_{x,t})I_t = \frac{(1 - \tau_{n,t})W_tH_t}{P_t} + \Pi_t + T_t + (1 - \tau_{k,t})r_tK_t \]

\[ -\frac{M_t - M_{t-1}}{P_t} = \frac{B_{t+1} - (1 + i_{t-1})B_t}{P_t} - \frac{S_tB_{t+1}^* - S_t\Psi_t(1 + i_{t-1}^*)B_t^*}{P_t}, \]

where the law of motion for capital satisfies

\[ K_{t+1} = (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t. \]

Here, \( I_t \) denotes gross capital investment, \( \tau_{x,t}, \tau_{k,t}, \) and \( \tau_{n,t} \) are investment, capital income and labor income taxes, \( r_t \) is the rental rate of capital, and the term \( \Phi\left(\frac{I_t}{K_t}\right) \) represents adjustment costs for investment. As before, the function \( \Phi(\cdot) \) is increasing and concave. As in the model of GGN, aggregate consumption \( C_t \) is a composite of both domestic and foreign goods given by (8). Here, households are the owners of the capital stock. In this case, households choose consumption, labor, money holdings, domestic and foreign bonds, investment and next period’s capital stock.

The representative firm produces goods in a competitive environment with a technology given by

\[ Y_t = \tilde{A}_tK_t^\alpha L_t^{1-\alpha}, \]

where \( L_t = H_t^\Omega H_t^{\epsilon(1-\Omega)} \) and \( \tilde{A}_t \) is the technology parameter for this particular economy. Here, firms choose household labor \( H_t \), entrepreneur’s labor \( H_t^\epsilon \), and capital stock \( K_t \) in order to maximize profits \( \Pi_t \):

\[ \Pi_t = \left(\frac{P_{t,t}}{P_t}\right)\tilde{A}_tK_t^\alpha L_t^{1-\alpha} - \frac{W_tH_t}{P_t} - \frac{W_t^\epsilon H_t^\epsilon}{P_t} - r_tK_t. \]

The rest of the associated prototype economy is similar to the GGN model. Accordingly, the foreign sector is still exogenously given by equations (A.1) and (A.2) in Appendix A.

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7 As noticed, GGN assumes a utility function separable in real money holdings. Such an assumption is convenient in the sense that the marginal rate of substitution between leisure and consumption is independent of money holdings, as required by the benchmark prototype economy. As in the GGN model, money holdings in the associated prototype economy only serve to pin down the nominal money stock.
Similarly, monetary policy instruments available are either the nominal exchange rate or the nominal interest rate.

As the associated prototype economy is now fully described, the following proposition shows that the detailed economy of GGN has the same allocations as those in the associated prototype economy.

**Proposition 1** Consider the prototype economy associated to the model of Gertler et al. (2006) with efficiency, labor, investment, and government consumption wedges given respectively by

\[
\begin{align*}
\bar{A}_t &= \omega_t A_t \omega_t^0, \\
1 - \tau_{n,t} &= \frac{\preuss^{C_t}}{(1 - \xi)(1 - H_t) F_{n,t}}, \\
\frac{1}{1 + \tau_{x,t}} &= \frac{1}{Q_t \left[1 + \chi \left(\frac{B_{t+1} + P_{i}}{N_{t+1}}\right)\right]}, \\
g_t &= C_t^{H*} + C_t^{eH} + G_t^H - \left(\frac{P_t P_t'}{P_{t+1}^H}\right) \left(C_t^F + I_t^F\right),
\end{align*}
\]

with

\[
\begin{align*}
P_{k,t} &\equiv \frac{1}{\Phi' \left(\frac{K_t}{K_{t+1}}\right)}, \\
\tilde{P}_{k,t} &\equiv P_{k,t} \left[1 - \delta + \Phi' \left(\frac{L_t}{K_t}\right) - \Phi' \left(\frac{L_t}{K_t}\right) \left(\frac{L_t}{K_t}\right)\right], \\
\tau_{k,t} &= \frac{\tau_{x,t} P_{k,t}}{r_t}, \\
r_t &= \frac{P_{W,t}}{P_t} F_{k,t}, \\
\frac{\tau_{n,t} W_{i,t} H_t}{P_t} + \tau_{k,t} r_t K_t + \tau_{x,t} I_t + \frac{M_{t-1} - M_{t-1}}{P_t} I_t - T_t &= G_t^H + C_t^{eH}.
\end{align*}
\]

Thus the equilibrium allocations of the model by Gertler et al. (2006) coincide with those of the associated prototype economy.

**Proof.** See Appendix A. ■

A general sketch of the proof is as follows. First consider the efficiency wedge. Substituting \(\bar{A}_t = \omega_t A_t \omega_t^0\) into the production function (23) leads to equation (11) in the GGN model. As described in detail in Appendix A, nominal price rigidities in the model of GGN cause a distortion between the marginal product of labor and real wages. As real wages are equal to the marginal rate of substitution between leisure and consumption in the GGN economy, a labor wedge of the form

\[
1 - \tau_{n,t} = \frac{\preuss^{C_t}}{(1 - \xi)(1 - H_t) F_{n,t}}
\]
allows to capture the distortions arising from nominal price rigidities. On the other hand, Appendix A also shows that the financial accelerator mechanism embedded in the function \( \chi \left( \frac{B_{t+1}/P_t}{N_{t+1}} \right) \) creates a wedge in an otherwise standard Euler equation. Such a distortion may be represented by the investment wedge

\[
\frac{1}{1 + \tau_{x,t}} = \frac{P_{k,t}}{Q_t \left[ 1 + \chi \left( \frac{B_{t+1}/P_t}{N_{t+1}} \right) \right]}. 
\]

Thus, from a business cycle accounting perspective the financial accelerator mechanism may be reinterpreted in terms of an investment wedge in the prototype economy. Finally, substituting the last two expressions of the proposition into the household’s budget constraint (21) recovers the government wedge.

3 Estimation and Results

This section presents a series of exercises to evaluate the result reported in Gertler et al. (2006) whereby financial frictions account for nearly half of the decline in economic activity during the 1997-1998 crisis in Korea. In particular, both the benchmark prototype economy and the result of Proposition 1 are used to seize the contribution of the investment wedge, either separately or in combinations, to fluctuations in output, investment and labor under the business cycle accounting method. For that purpose, the estimation method is explained first. Next, it is shown that, according to the business cycle accounting method, the result of GGN hinges on the relatively small value for the Tobin’s \( q \) elasticity. This finding is supported after simulating the benchmark prototype economy using larger values for the Tobin’s \( q \) elasticity. In these scenarios, the contribution of the investment wedge for explaining macroeconomic fluctuations decreases as the elasticity is arguably more in accord with empirical evidence. Finally, a sensitivity analysis is performed by allowing for variable capital utilization and measurement errors in the benchmark prototype economy. In either case, it is still found that the contribution of the investment wedge in isolation to economic fluctuations is sensitive to the value of Tobin’s \( q \) elasticity.

3.1 Estimation method

The accounting procedure of Chari et al. (2006a) may be implemented in two steps. First, wedges of the benchmark prototype economy are measured by using both the data and a
detrended version of the model. Then the prototype model is simulated using the wedges already obtained to assess the contribution of wedges (either separately or in combinations) to fluctuations in the variables of interest such as output, labor and investment.

Measurement of wedges

As in Chari et al. (2006a), standard functional forms for preferences and technology are assumed to measure the wedges in the benchmark prototype model. In particular, preferences are of the logarithmic form \( U(c, l) = \log c + \psi \log(1 - l) \) and the production function is of the Cobb-Douglas type \( F(k, l) = k^{\alpha}l^{1-\alpha} \). Some parameters of the model are calibrated as in the business cycle literature and some others are estimated as follows. First, the series for the capital stock is constructed by using the law of motion (1), given data on investment and an initial choice of the capital stock \( k_0 \). As equations (2) - (5) conveniently describe the equilibrium of the benchmark prototype economy, consumption \( c_t \) from the resource constraint (3) may be substituted into (4) and (5) so that a system of three equations (2), (4) and (5) in three unknowns (output, labor, and investment) may be log-linearized. Now define a vector \( s_t \) for the four wedges, \( s_t = (\log A_t, \tau_{nt}, \bar{\tau}_{xt}, \log g_t) \), that follows a vector autoregressive AR(1) process of the form

\[
s_{t+1} = P_0 + Ps_t + \varepsilon_{t+1},
\]

where the innovation \( \varepsilon_t \) is i.i.d. and distributed normally with mean zero and covariance matrix \( V \). Here, a lower-triangular matrix \( Q \) such that \( V = QQ' \) is estimated to ensure that the resulting \( V \) is positive semidefinite. Thus the economy is defined by a system of seven equations, three from equilibrium conditions and four from the AR(1) process for the wedges. The four wedges are a function of the history of events up through and including period \( t \) (i.e., the state). Parameters included in matrices \( P_0, P \) and \( V \) of the AR(1) process for the wedges are then estimated using maximum likelihood methods.⁸

Once the stochastic process in (25) is estimated, the four wedges may be recovered from the data and the equilibrium conditions of the benchmark economy. For example, the government consumption wedge may be measured directly from the data as the sum of government expenditures and net exports. The efficiency and labor wedges may be obtained directly from equations (2) and (4), given a series for the capital stock \( k_t \). Finally, the investment wedge is recovered from expression (5). A potential problem is that the investment wedge

⁸ See Chari et al. (2006b) for details.
cannot be directly obtained from (5) as this requires to specify expected values over future consumption, capital stock and wedges. Accordingly, let \( y_t^d, l_t^d, x_t^d \), and \( k_0^d \) denote data on production, labor, investment and the initial capital stock, respectively, and let \( y(s_t, k_t), l(s_t, k_t), \) and \( x(s_t, k_t) \) represent the decision rules of the model. Then the realized wedge series \( s_t^d \) solves

\[
y_t^d = y(s_t^d, k_t), \quad l_t^d = l(s_t^d, k_t), \quad x_t^d = x(s_t^d, k_t),
\]

with \( k_{t+1} = (1 - \delta)k_t + x_t^d - \varphi \left( x_t^d / k_t \right) k_t, \quad k_0 = k_0^d \) and \( g_t = g_t^d \). As detailed in Chari et al. (2006b), these decision rules may be combined with the estimated stochastic process for the wedges in order to obtain the series for the realized wedge \( \tilde{\tau}_{x,t} \).

**Wedge decomposition**

Once the realized sequence of wedges is obtained, the benchmark prototype model may be simulated in order to assess, separately and in combinations, the contribution of wedges to fluctuations in variables of interest starting at some initial date. This contribution is measured by comparing the realizations of variables such as output, labor and investment arising from simulating the model to those in the data. For example, define the efficiency wedge component as the vector of wedges \( s_{1t} = (\log A_t, \tau_{n,t}, \tau_x, \log \bar{g}) \) so that in period \( t \) the efficiency wedge takes on its period \( t \) value while simultaneously keeping the other wedges at some constant values. The corresponding decision rules may be denoted by \( y_e(s_{1t}, k_t), l_e(s_{1t}, k_t), \) and \( x_e(s_{1t}, k_t) \). These decision rules along with an initial condition \( k_0^d \), the realized wedge series \( s_t^d \) and the law of motion for capital may be used to compute the realized sequence of output, labor and investment, denoted respectively by \( y_e^t, l_e^t \) and \( x_e^t \). These results may be then directly compared to actual data. Naturally, this accounting exercise may be performed in alternative ways, given the corresponding definitions for the labor wedge component \( s_{2t} \), the investment wedge component \( s_{3t} \), and the government consumption wedge component \( s_{4t} \). It is also possible to construct series for combined wedges. For example, the efficiency plus labor component may be defined as \( s_{5t} = (\log A_t, \tau_{n,t}, \tau_x, \log \bar{g}) \). If the four wedges are fed into the decision rules in (26) and used in combination with both the law of motion for capital and the equation \( \log g_t(s_t^d) = \log g_t \), all the movements in output, labor and investment from the simulation are exactly those observed in the data by construction.
3.2 Calibration and Results

3.2.1 Calibration

Before implementing the business cycle accounting method, it is required to set functional forms for technology, preferences, and the adjustment cost function, as well as parameter values as in the real business cycle literature. As mentioned earlier, for the benchmark prototype economy the production function is given by \( F(k_t, l_t) = k_t^{\alpha}l_t^{1-\alpha} \), and preferences are of the logarithmic form \( U(c_t, l_t) = \log c_t + \psi \log(1 - l_t) \).\(^9\) The adjustment cost function is specified by \( \phi(x/k) = (a/2)(x/k - b)^2 \), which is commonly used in the literature, as in Chari et al. (2006a). The capital share parameter \( \alpha \) is fixed to 0.31, which is roughly consistent with the estimates for Korea reported in Gollin (2002) and Young (1995). The annual depreciation rate \( \delta \) is set to 6 percent. Even though this parameter value might seem low, it is well within the interval reported by Lubik and Teo (2005) using Bayesian estimation techniques for a sample of both developed and developing countries. The time allocation parameter \( \psi \) is set to 2.95 so that Korean households allocate about one third of their time to working activities, consistent with data reported by the Korean National Statistical Office. Population growth \( n \) and exogenous technology growth \( \gamma \) are set to 1.5 and 5 percent on an annualized basis, to be consistent with Korean data for the period 1982:3 - 2005:2. Finally the discount factor \( \beta \) is 0.99.

In reference to the adjustment cost function, parameter \( b \) is set equal to the investment-capital share at the steady state, namely \( b = (1 + n)(1 + \gamma) - 1 + \delta \), so that adjustment costs are zero at the steady state. The value for parameter \( a \) is set to be consistent with a series of alternative values for the elasticity of investment-to-capital ratio with respect to the price of capital (henceforth “Tobin’s q elasticity”). In particular, such an elasticity in the model is defined by

\[
\eta \equiv \frac{d \log(x_t/k_t)}{d \log P_{k',t}} = \frac{1}{b\phi''},
\]

where \( P_{k,t} \) is the market price of capital in the benchmark prototype economy determined by \( P_{k',t} = 1/(1 - \phi'(x_{t+1}/k_{t+1})) \). Thus given the values for both \( b \) and Tobin’s \( q \) elasticity, a value for \( a \) may be recovered.

\(^9\) Chari et al. (2006a) show that the business cycle accounting method is qualitatively robust to alternative specifications of production functions and preferences.
Setting an appropriate value for Tobin’s $q$ elasticity in these models is sometimes controversial. Using Bayesian techniques, Elekdag et al. (2006) find a point estimate of 2.39 for Tobin’s $q$ elasticity using Korean data, with 90 percent probability bands from 1.9 to 3.12. Lubik and Teo (2005) report point estimates between 1.76 and 3.23 using Bayesian techniques in a sample of both developed and developing countries. As a reference, Gertler et al. (2006) arbitrarily set $\eta = 1/2$. Bernanke et al. (1999) argue that reasonable assumptions about adjustment costs suggests that the inverse of Tobin’s $q$ elasticity should lie within a range from 0 to 0.5 as higher values would yield implausibly high adjustment costs. This reasoning is also shared by Chari et al. (2006a), although Christiano and Davis (2006) argue that this conclusion is based on the assumption that the adjustment cost function is globally quadratic. Given the controversy about reasonable values for Tobin’s $q$ elasticity, the business cycle accounting exercise considers the alternative values of 0.5, 1, 3 and $\infty$. A value of 0.5 is justified as this is the value used by Gertler et al. (2006). An elasticity of 1 is favored by Christiano and Davis (2006) whereas an elasticity of 3 roughly reflects the highest value in the interval reported by Elekdag et al. (2006) using Korean data. Finally, a value of infinity is consistent with the assumption of zero adjustment costs in the benchmark prototype economy.

The model is estimated using quarterly data for Korean output, hours, investment, and government expenditures (including the external sector) for the period 1982:3 - 2005:2 (details about data sources and construction of variables can be found in Appendix C). For convenience, both output and labor are normalized to 100 for the base period 1997:4, as this is the quarter in which the crisis began. In the figures shown below, investment is divided by the base period level of output. As a final remark, earlier versions of Chari et al. (2006a) allowed for the possibility of measurement errors in the state-space form of the benchmark prototype economy. For simplicity, measurement errors are set to zero in the first part of the exercise. Later this assumption is relaxed as Christiano and Davis (2006) report that results may be sensitive to measurement errors.

3.2.2 Results

The business cycle accounting exercise is focused on the period 1997:4 - 2000:2 as it comprises the Korean crisis period. The properties of the wedges using the whole sample (1982:3 - 2005:2) are described in Appendix B. Table 1 presents the estimated values for wedges and
parameters $P$ and $Q$ in equation (25), as well as their corresponding 90 percent confidence intervals for a Tobin’s $q$ elasticity value of 0.5.\footnote{As detailed in Chari et al. (2006b), the confidence intervals are estimated from a bootstrapped distribution with 500 replications.} Here, a Tobin’s $q$ elasticity of 0.5 is initially assumed as this is the value reported by GGN. Figure 1 shows actual Korean output as well as the series for efficiency, labor and investment wedges from the benchmark prototype economy, represented by $A_t, (1 - \tau_n)$ and $1/(1 + \tilde{\tau}_x)$, respectively. For this period, it may be observed that the labor and investment wedge fall about 17 and 43 percent in the first quarter of 1998 respectively, and exhibit a relatively rapid recovery. The efficiency wedge depicts a slightly downward trend during the next two years following the start of the crisis.

Figure 2 displays data on output as well as for the investment wedge under alternative values of Tobin’s $q$ elasticity. It may be observed that, as the elasticity goes from 0.5 to infinity, the contemporaneous correlation between output and the investment wedge goes from positive to negative in general. The intuition for this result is relatively straightforward. Recall that the investment wedge in the benchmark prototype economy is defined by $1 + \tilde{\tau}_{x,t} \equiv \frac{1 + \tau_{x,t}}{1 - \varphi'(x_t/k_t)}$. If adjustment costs are absent from the model, then $\tilde{\tau}_{x,t} = \tau_{x,t}.$ In such a case, the investment wedge roughly moves in opposite direction to output in the data as illustrated in Figure 2. Now consider adjustment costs. In recessions, when $x_t/k_t$ is relatively low, the expression $\varphi'(x_t/k_t)$ is negative. Therefore, $\tilde{\tau}_{x,t}$ is positive as long as $\tau_{x,t}$ is larger than (the absolute value of) $\varphi'(x_t/k_t)$. This is roughly satisfied for small values of the parameter $a$ (i.e., for relatively large elasticity values). As the elasticity decreases ($a$ increases), there is some point at which the value for $\tilde{\tau}_{x,t}$ becomes negative. In such a case, the investment wedge is now positively correlated with output in the data. Thereafter, specifying a higher level of adjustment costs causes the investment wedge to fall even further during recessions.

Now Figure 3 shows data on output, labor and investment and their corresponding simulations when only the investment wedge is in place while the rest of wedges are turned off (i.e., simulations due to the investment wedge component). The results are presented for alternative values of Tobin’s $q$ elasticities. First consider the case where $\eta = 0.5$ as this is the value used by GGN in their model. In such a case, the investment wedge alone accounts for about half of the fall in actual output registered in the first quarter of 1998 and at the trough of the recession, and consistently explains a substantial fraction of such a fall during the period of analysis. A similar qualitative result is found for investment. These
findings are remarkable as these are precisely the results reported by GGN.\textsuperscript{11} As for labor, the investment wedge is also capable of substantially explain its fall and recovery for the first five quarters after the crisis began, but predicts a slower increase thereafter as compared to actual data.\textsuperscript{12}

Consider now the case where the elasticity slightly increases to one. The fall in output due to the investment wedge alone is now lower. In fact, the investment wedge only explains about 41 percent of the fall in actual output during the first quarter of 1998 (37 percent at the trough of the recession). This lower fall in simulated output is due to the lower fall in both simulated labor and investment during the period of analysis. If Tobin’s \(q\) elasticity increases to 3, the investment wedge only explains about 16 percent (19 percent) of the fall in output registered in the first quarter of 1998 (at the trough of the recession). Finally, if investment adjustment costs were absent (alternatively, if elasticity increases to infinity) the investment wedge alone would increase output from its initial value right after the crisis and for subsequent periods in general. This inconsistency with the actual data also holds for labor and investment.\textsuperscript{13}

An alternative to seize the importance of the investment wedge to explain movements in output, labor and investment is to evaluate the response of these variables should the investment wedge be the only one absent from the model. The results are presented in Figure 4 under alternative Tobin’s \(q\) elasticities. As before, consider first the case where elasticity is 0.5. In such a case, efficiency, labor and government wedges altogether fall short from explaining the decrease in output. This is also true for labor and investment.\textsuperscript{14} In other words, the investment wedge is crucial in explaining fluctuations if the elasticity is 0.5. However, this relative importance decreases as Tobin’s \(q\) elasticity increases. For example, if

\begin{itemize}
  \item \textsuperscript{11} Gertler et al. (2006) do not report results on labor.
  \item \textsuperscript{12} The relatively rapid recovery of labor observed in the data might be explained by the series of reforms introduced in early 1998 to make labor markets more flexible. For details, see Koo and Kiser (2001).
  \item \textsuperscript{13} The investment wedge component also exhibits the consumption anomaly reported in Chari et al. (2006a) in the sense that both simulated investment and private consumption are negatively correlated, in contrast with the positive correlation found in the data. This result holds for alternative elasticity values and for each of the specifications detailed below (i.e., variable capital utilization and measurement error).
  \item \textsuperscript{14} Otsu (2006) considers a roughly standard small open economy model to account for the Korean crisis. He finds that technology shocks (the efficiency wedge in the prototype economy) can reasonable explain fluctuations in output, labor and investment. However, his model exhibits the stochastic singularity problem discussed by Ingram et al. (1994) and Ireland (2004). Therefore, a direct comparison between the results in Otsu (2006) and those reported here is not possible.
\end{itemize}
the elasticity is 3 then the investment wedge would play a smaller role in output fluctuations, or even a virtually no role if the elasticity is somewhere between 3 and infinity.

It is worth mentioning that the sensitivity of the investment wedge component to alternative elasticity values also holds for the labor wedge component. Figure 5 presents data on output, labor and investment as well as the predictions of the model with just the labor wedge. If the elasticity is 0.5, the labor wedge component replicates data on labor extremely well for the whole period, but it falls short from explaining investment. As a consequence, the simulated fall in output cannot replicate the fall in actual output. If the elasticity goes to infinity, a different result is obtained: the fall in output, labor and investment are overestimated for several periods. As for the efficiency and government wedge components, the predictions for macroeconomic variables are relatively less affected by alternative Tobin’s q elasticities.\footnote{Similar qualitative results are found under the variable capital utilization and measurement errors economies described below. In particular, the labor wedge component remains sensitive to the parameterization of the adjustment cost but neither the efficiency nor the government wedge component exhibit substantial changes.}

3.3 Sensitivity Analysis

3.3.1 Variable Capital Utilization

A potential drawback of the business cycle accounting exercise presented above is that it assumes a fixed capital utilization. This in principle may be a problem as GGN consider a model with variable capital utilization (see equation 11). The idea is that by introducing variable capital utilization into the benchmark prototype economy, the measurement of the efficiency wedge may be directly affected. This change may potentially affect the measurement of all other wedges and thus the relative contribution of wedges to macroeconomic fluctuations.

To address this issue, the specification of Chari et al. (2006a) is followed. In particular, suppose that the production function in the benchmark prototype economy is replaced by

$$y_t = A_t(k_t h_t)^{\alpha}(\bar{n} h_t)^{1-\alpha}$$

where $\bar{n}$ is the number of workers employed and $h_t$ is the length of the workweek. Hence total labor input is $l_t = \bar{n} h_t$. If the number of workers is assumed to be constant, all the variation in labor arises from the workweek $h_t$. In such a case, the services of capital, $k_t h_t$, are proportional to the product of the stock $k_t$ and labor input $l_t$, so that the flow of capital services is affected by variations in the labor input $l_t$. Under
this interpretation, the number of workers employed $\bar{n}$ may be normalized to 1 so that the production function in the benchmark prototype economy is now given by

$$y_t = A_t k_t^\alpha l_t.$$  

Figure 6 presents data and simulated movements in variables with just the investment wedge assuming variable capital utilization. As before, alternative values for Tobin’s $q$ elasticity are considered. When elasticity is 0.5, the investment wedge alone explains about 60 percent (48 percent) of the decline in output observed in the first quarter of 1998 (at the trough of the recession). Again, this result is remarkable as it coincides with the findings of GGN. However, as the elasticity increases the investment wedge roughly explains a lower fraction of the fall in all the variables. For example, it now only accounts for about six percent (11 percent) of the output fall in 1998:1 (at the trough of the recession) when elasticity is 3, as compared to about 16 percent (19 percent) in the model with fixed capital utilization. In fact, increasing the elasticity to a value of infinity now yields the opposite results: all the three variables due to the investment wedge increase from their corresponding initial values, a result completely at odds with the data.

As before, an alternative exercise is to exclude the investment wedge from the analysis while keeping the other wedges in place. Results are presented in Figure 7. If the elasticity is set to 0.5, the result is the same as before: investment wedges are important for explaining the data. However, it becomes evident that if the elasticity is around 3, the investment wedge is irrelevant in accounting for movements in output, labor and investment. As the elasticity goes to infinity, ignoring the investment wedge leads to larger falls in all the three variables for several periods as compared to the data. In such a case, the large increase in output, labor and investment from the investment wedge alone (already shown in Figure 6) is needed to allow for a better match with the data.

### 3.3.2 Measurement Errors

In their study on the business cycle accounting method, Christiano and Davis (2006) find that the results of this methodology may be sensitive to small changes in the specification of measurement errors. Taking this observation into account, now the benchmark prototype
The economy of section 2 is modified to include measurement errors. In general, it may be shown that the state-space form of the benchmark prototype economy may be described by the system

\[ X_{t+1} = AX_t + B\epsilon_{t+1}, \]

\[ Y_t = CX_t + \omega_t, \]

and

\[ \omega_t = D\omega_{t-1} + \zeta_t, \]

where \( X_t \) is a vector of states, \( Y_t \) is a vector of observables, \( \omega_t \) is a \( 4 \times 1 \) vector of measurement errors with covariance matrix \( E\zeta_t\zeta_t' = R \) and the property \( E\epsilon_t\zeta_s' = 0 \) for all periods \( t \) and \( s \), and matrices \( A, B \) and \( C \) are functions of parameters (see Chari et al. (2006b) for details). Elements of matrix \( D_{4 \times 4} \) are the parameters that describe the serial correlation of the measurement error. In the previous exercises, it was assumed \( D = 0_{4 \times 4} \) and \( R = 0 \times I_{4 \times 4} \) where \( I \) is the identity matrix. Following Christiano and Davis (2006), \( R \) is now set to \( R = 0.0001 \times I_{4 \times 4} \) while keeping \( D = 0_{4 \times 4} \).

Results due to the investment wedge component under the new parametrization of measurement errors are presented in Figure 8. As it may be noticed, the investment wedge component continues to be very sensitive to the parametrization of adjustment costs. For example, it accounts for about 48 percent of the output fall in the first quarter of 1998 (43 percent at the trough of the recession) when the elasticity is 0.5, and about 36 percent if the elasticity is slightly increased to one (35 percent at the trough of the recession). Higher values of Tobin’s \( q \) elasticity yield an increase in output (relative to its initial value) for several periods, in contrast with the data. This is also true for both labor and investment.

Finally, Figure 9 presents simulation exercises when only the investment wedge is cancelled while keeping the other three wages in place. If the elasticity value of 3 is considered, the investment wedge is not crucial for explaining fluctuations in output, labor and investment. However, this is no longer true for other elasticity values.

16 Measurement errors may be interpreted as all movements and co-movements in the data that the benchmark prototype economy is not capable of accounting for, as in Ireland (2004).
4 Conclusions

In their quantitative exercise to explain fluctuations in macroeconomic variables for Korea during the crisis of 1997-1998, Gertler et al. (2006) find that the financial accelerator mechanism may account for nearly half of the decline in Korean output and investment during that period. In this paper, the model of Gertler et al. (2006) is studied from the business cycle accounting perspective of Chari et al. (2006a) to seize the importance of the financial accelerator in explaining macroeconomic fluctuations.

The paper is divided in two parts. In the first part, the relevant models (i.e., the benchmark prototype economy, the model of Gertler et al. (2006) and the associated prototype economy) are presented along with an equivalence result. The equivalence result consists in showing that the model of Gertler et al. (2006) may be represented by a prototype economy with four wedges: efficiency, labor, investment and government consumption wedges. In particular, it is shown that the financial accelerator mechanism of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Bernanke et al. (1999), among others, may be captured by an investment wedge. In the second part, the benchmark prototype economy with adjustment costs is used to recover the series of all four wedges from Korean data and see whether the investment wedge, either separately or in combinations with other wedges, can consistently explain the drop in output and other macroeconomic variables during the Korean crisis of 1997-1998.

Remarkably, the business cycle accounting method is capable of reproducing the quantitative results found in Gertler et al. (2006). Namely, it is found that the investment wedge component may explain about half of the decline in Korean output and investment assuming the same Tobin’s $q$ elasticity value as in Gertler et al. (2006). However, such a value yields precisely the highest contribution of the investment wedge in explaining output fluctuations among the values considered in this paper. If the elasticity increases slightly, the investment wedge component becomes less important and may even yield results exactly opposite to those found in the data. Hence the contribution of the investment wedge for explaining the drop in Korean output is very sensitive to the parametrization of the adjustment cost function. This result is robust to alternative specifications of the benchmark prototype economy, including variable capital utilization and measurement errors.

It is worth mentioning that assuming a relatively large elasticity does not necessarily neglect the importance of financial frictions in trying to explain the Korean crisis. There
are at least two reasons that lead to such a warning. First, if only the investment wedge is excluded from the analysis while maintaining the other three wedges in place, the data cannot be matched satisfactorily under some parametrizations. This suggests that the investment wedge may be important for explaining macroeconomic fluctuations, although this result is also sensitive to alternative values for Tobin’s $q$ elasticity. Second, it is possible that financial frictions may be represented by other wedges in the prototype model. For example, Chari et al. (2006a) present an economy where financial frictions are captured by the efficiency wedge in the benchmark prototype economy.

Finally, and most importantly, this paper illustrates that the parametrization of the adjustment cost function may play an important role in evaluating the relative importance of a particular wedge to explain business cycle fluctuations. The exercises presented above consider a quadratic adjustment cost function, as is typically assumed in the literature. As previously discussed, both labor and investment wedges are particularly sensitive to alternative values for Tobin’s $q$ elasticities. Thus for some elasticity values the investment wedge is more important than the labor wedge to explain fluctuations, but not for others in Korean data. Evidence for Korea suggests that an appropriate value for Tobin’s $q$ elasticity should lie between 1.9 and 3. These values are roughly within the interval of 2 to infinity suggested by Bernanke et al. (1999) and Chari et al. (2006a), among others, as this interval imply small adjustment costs. If one is willing to accept these values, then the labor wedge would be apparently more important than the investment wedge to explain the Korean crisis.

Nevertheless, Kydland and Prescott (1982) argue that a quadratic adjustment cost function is inconsistent with the data in the sense that it implies that the elasticity of the investment-capital ratio with respect to the price of capital is the same in the short run and the long run. In this regard, Christiano et al. (2005) and Christiano and Davis (2006) present an alternative adjustment cost function that avoids the criticisms of Bernanke et al. (1999), Chari et al. (2006a), and Kydland and Prescott (1982). Using this alternative function, Christiano et al. (2005) report a point estimate for Tobin’s $q$ elasticity of 0.4, which is slightly smaller than the value adopted by Gertler et al. (2006). Unfortunately, this value cannot be directly compared with available estimates of investment elasticities, as steady-state deviations of investment in the model of Christiano et al. (2005) depend on its own lag as well as on contemporaneous and future prices of capital. In this regard, it might be interesting to find empirical counterparts for Tobin’s $q$ elasticity consistent with
the adjustment cost function of Christiano et al (2005). This is an issue that deserves further analysis as this elasticity value may yield radically different results about the importance of particular wedges (and thus distortions for a large class of models) in trying to explain macroeconomic fluctuations.

References


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Appendix A

This appendix presents the foreign sector, the retailer’s problem and the first-order conditions of the model by Gertler et al. (2006). It also describes the first-order conditions of the associated prototype economy and shows in detail the proof of Proposition 1 presented in the main text.

The financial accelerator model of Gertler et al. (2006)

Foreign Sector. In the model, the foreign sector is exogenously described by the price of tradable foreign goods and by foreign consumption of tradable domestic goods. For simplicity, the law of one price holds for tradable foreign goods at the wholesale level. If \( P_{Ft} \) denote the foreign currency price of foreign goods and \( P_{W,t}^{F*} \) is the wholesale price of foreign goods in domestic currency, it must be the case that

\[
P_{W,t}^{F} = S_{t} P_{t}^{F*}. \tag{A.1}
\]

Foreign demand for the home tradable good, \( C_{Ht}^{*} \), is given by

\[
C_{Ht}^{*} = \left[ \left( \frac{P_{Ht}^{*}}{P_{t}^{*}} \right)^{-\kappa} Y_{t}^{*} \right]^{v} (C_{t-1}^{H*})^{1-v}, \quad 0 \leq v \leq 1, \tag{A.2}
\]

where \( Y_{t}^{*} \) is (exogenous) real foreign output. The expression \( (C_{t-1}^{H*})^{1-v} \) allows for inertia in foreign demand for domestic products, and may arise from foreign preferences with habit formation. Finally, the foreign gross nominal interest rate \((1 + i_{t}^{*})\) and the nominal price (in units of foreign currency) of the foreign tradable good, \( P_{t}^{F*} \), are taken as exogenous.

Retailers. There is a continuum of monopolistically competitive retailers in the unit interval. Retailers buy wholesale goods from entrepreneurs in order to resale these goods to households, capital producers, government and the foreign country after a slight differentiation of the product at a fixed cost \( \kappa \). Let \( Y_{t}^{H}(z) \) be the good sold by retailer \( z \). The final domestic good is assumed to be a CES composite of individual retail goods:

\[
Y_{t}^{H} = \left[ \int_{0}^{1} Y_{t}^{H}(z)^{\frac{1}{\sigma-1}} dz \right]^{\frac{\sigma-1}{\sigma}} - \kappa. \tag{A.3}
\]

Let \( P_{t}^{H} \) denote the price of the composite final domestic good. This price is given by

\[
P_{t}^{H} = \left[ \int_{0}^{1} P_{t}^{H}(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}. \tag{A.4}
\]
Cost minimization implies that the demand faced by retailer \( z \) is given by \( Y_t^U(z) = \left( \frac{P_{W,t}^U(z)}{P_t^U} \right)^{-\theta} Y_t^U \). The marginal cost of producing a unit of output is the relative wholesale price \( \frac{P_{W,t}^U}{P_t^U} \) as retailers simply repackage wholesale goods.

Retailers set nominal prices on a staggered basis, as in Calvo (1983). As is well known, in such a framework at time \( t \) some retailers may reset their price optimally with probability \((1 - \theta)\), regardless of the time elapsed since the last adjustment. Thus the parameter \( \theta \) captures the degree of price stickiness in the economy. These retailers choose prices to maximize expected discounted profits subject to the demand of their own good and the constraint on the frequency of price adjustments. Let \( \bar{P}_t^H \) denote the optimal price chosen for all retailers setting their price at time \( t \). Around the steady state, such an optimal price is simply

\[
\bar{P}_t^H = \mu \prod_{i=0}^{\infty} (P_{W,t+i})^{(1-\beta\theta)(\beta\theta)}.
\]

(A.5)

Here, \( \mu \equiv \frac{1}{1-1/\theta} \) is the retailer’s desired gross mark-up over wholesale prices. In a world of perfectly flexible prices, equation (A.5) simply reduces to \( \bar{P}_t^H = \mu P_{W,t} \).

Given the optimal price \( \bar{P}_t^H \), the domestic price index in the neighborhood of the steady state is given by

\[
P_t^H = (P_{t-1}^H) \theta \left( \bar{P}_t^H \right)^{1-\theta}.
\]

(A.6)

A combination of expressions (A.5) and (A.6) yields the so-called new Phillips curve for the price of domestic final goods:

\[
\frac{P_t^H}{P_{t-1}^H} = \left( \frac{\mu P_{W,t}}{P^H_t} \right)^{\lambda} E_t \left\{ \frac{P_{t+1}^H}{P_{t}^H} \right\}^{\beta},
\]

with \( \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \). In a similar vein, foreign goods in the domestic economy are subject to a Calvo-style price setting environment as in the domestic goods case. It may be shown that if retailers face the marginal cost \( P_{W,t}^F \), the inflation rate for foreign goods is

\[
\frac{P_t^F}{P_{t-1}^F} = \left( \frac{\mu F_t P_{t}^F}{P_t^F} \right)^{\lambda_f} E_t \left\{ \frac{P_{t+1}^F}{P_{t}^F} \right\}^{\beta},
\]

(A.8)

where \( \lambda_f \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \) and \( \theta_f \) denotes the degree of price stickiness for foreign goods.
Finally, given that the consumer price index $P_t$ is a function of both domestic and foreign good prices, the CPI inflation rate around the steady state may be written as

$$\frac{P_t}{P_{t-1}} = \left( \frac{P^H_t}{P^H_{t-1}} \right)^\gamma \left( \frac{P^F_t}{P^F_{t-1}} \right)^{1-\gamma}. \quad (A.9)$$

**First-order Conditions.** The problem of the household in the model of GGN is to maximize (6) subject to (7) and the budget constraint (10). Optimality conditions of the problem are given by

$$\frac{C^H_t}{C^F_t} = \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{P^H_t}{P^F_t} \right)^{-\rho}, \quad (A.10)$$

$$(1-\zeta)(C_t)^{(\sigma-1)(\sigma-1)-1}(1-H_t)^{\zeta(1-\gamma)} = \lambda_t, \quad (A.11)$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} (1+i_t) \frac{P_t}{P_{t+1}} \right\}, \quad (A.12)$$

$$\frac{\xi}{M_t} = \lambda_t \frac{1}{P_t} - \beta E_t \left\{ \lambda_{t+1} \frac{1}{P_{t+1}} \right\}, \quad (A.13)$$

$$E_t \left\{ \lambda_{t+1} \frac{P_t}{P_{t+1}} \left[ (1+i_t) - \Psi_t (1+i_t^*) \frac{S_{t+1}}{S_t} \right] \right\} = 0, \quad (A.14)$$

and

$$\frac{\varsigma C_t}{(1-\zeta)(1-H_t)} = \frac{W_t}{P_t}. \quad (A.15)$$

The first equation is derived from the minimization cost problem of the household. The remaining equations are roughly standard, where $\lambda_t$ is the marginal utility of consumption and $(1+i_t) \frac{P_t}{P_{t+1}}$ is the gross real interest rate. As the monetary policy instrument is defined in terms of either the nominal exchange rate or the nominal interest rate in the model of GGN, equation (A.13) is only useful to pin down the nominal money stock. The term in brackets in expression (A.14) is the uncovered interest parity condition.

Wholesale producers maximize gross output (12) net of labor and capital utilization costs, given the production function (11). The corresponding optimality conditions are summarized by:
\[(1 - \alpha) \Omega \frac{Y_t}{H_t} = \frac{W_t}{P_{W,t}}, \quad \text{(A.16)}\]

\[(1 - \alpha)(1 - \Omega) \frac{Y_t}{H^c_t} = \frac{W^e_t}{P_{W,t}}, \quad \text{(A.17)}\]

and

\[\alpha \frac{Y_t}{u_t} = \delta'(u_t) K_t \frac{P_{I,t}}{P_{W,t}}, \quad \text{(A.18)}\]

To capture the delayed response of investment in the data, capital producers make their production plans one period in advance. The corresponding problem is to

\[
\max_{K_{t-1}, l^n_t} \left\{ P_t Q_t \Phi \left( \frac{I^n_t}{K_t} \right) K_t - P_{I,t} I^n_t - P_{r,t} l^n_t K_t \right\}
\]

with first-order conditions

\[E_{t-1} \left\{ \Phi' \left( \frac{I^n_t}{K_t} \right) Q_t - \frac{P_{I,t}}{P_t} \right\} = 0, \quad \text{(A.19)}\]

and

\[r^l_t = E_{t-1} \left\{ Q_t \left[ \Phi \left( \frac{I^n_t}{K_t} \right) - \Phi' \left( \frac{I^n_t}{K_t} \right) \frac{I^n_t}{K_t} \right] \right\}, \quad \text{(A.20)}\]

where \(r^l_t\) represents the leasing rate of capital.

**The associated prototype economy**

In the associated prototype economy, households maximize lifetime utility (6) subject to (7), (21) and (22) for a given initial level of capital, domestic and foreign bonds. In such a case, optimality conditions are described by (A.10) - (A.14) along with

\[
\frac{SC_t}{(1 - \zeta)(1 - H_t)} = (1 - \tau_{n,t}) \frac{W_t}{P_t}, \quad \text{(A.21)}
\]

and

\[(1 + \tau_{x,t}) \lambda_t P_{k,t} = \beta E_t \left\{ \lambda_{t+1} \left[ (1 - \tau_{k,t+1}) r_{t+1} + (1 + \tau_{x,t+1}) \tilde{P}_{k,t+1} \right] \right\}, \quad \text{(A.22)}\]

where
\[ P_{k,t} = \frac{1}{\Phi'(\frac{I_t}{K_t})} \]

and

\[ \tilde{P}_{k,t} = P_{k,t} \left[ 1 - \delta + \Phi'\left(\frac{I_t}{K_t}\right) - \Phi'\left(\frac{I_t}{K_t}\right) \left(\frac{I_t}{K_t}\right) \right]. \]

Firms in the associated prototype economy maximize profits (24). First-order conditions are given by (A.16), (A.17) and

\[ \alpha Y_t \left(\frac{P_{W,t}}{P_t}\right) = r_t K_t. \]

**Proof of Proposition 1**

To prove the proposition described in the main text, consider first the efficiency wedge. Let such a wedge be given by \( \Delta_t = \omega_t \omega_t u_t^a \). Substituting this expression into (23) leads to the production function (11) in the GGN model.

Next, consider the labor wedge \( 1 - \tau_{n,t} \). The idea is to show that nominal price rigidities in the GGN model may be represented in terms of a labor wedge in the associated prototype economy. For such a purpose, it is useful to write down the maximization problem for those retailers that are allowed to set its price optimally at time \( t \):

\[
\max_{P_t} E_t \sum_{i=0}^{\infty} \theta \Delta_{t+i} \left( \left( \frac{P_{H,t}(z)}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{P_{H,t}(z)}{P_{t+i}} \right)^{-\theta} \right) Y_{t+i}^H,
\]

where \( \Delta_{t+i} \) is the discount factor, \( Y_{t+i}^H(z) = \left( \frac{P_{H,t}(z)}{P_{t+i}} \right)^{-\theta} \) \( Y_t^H \) represents the isoelastic demand faced by each retailer, and \( \varphi_t^r \) is the real marginal cost. Following GGN, as retailers simply repackaged wholesale goods at no extra cost, the marginal cost of an extra unit of output is just the relative wholesale price \( \frac{P_{W,t}}{P_t} \). Since the wholesale sector is perfectly competitive, the marginal cost for wholesale producers, \( \varphi_t^w \), must be equal to the relative wholesale price \( \frac{P_{W,t}}{P_t} \). Hence it must hold that \( \varphi_t^r = \varphi_t^w \), where it may be shown that the marginal cost \( \varphi_t^w \) is a function of the relative price of investment \( \frac{P_{I,t}}{P_t} \), and real wages for entrepreneurs and households denoted by \( \frac{W_t^e}{P_t} \) and \( \frac{W_t}{P_t} \), respectively.

It follows that the optimal price \( P_{t}^H \) set by those retailers allowed to change prices at time \( t \) is given by
\[
\frac{P_t^H}{P_t^H} = \left( \frac{\partial}{\partial - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \theta^i \Delta_i H_{t+i} \varphi^w_t (\frac{r^H_{t+i}}{P_t^{H}})^{\frac{\partial}{\partial - 1}} Y_t^H}{E_t \sum_{i=0}^{\infty} \theta^i \Delta_i H_{t+i} (\frac{r^H_{t+i}}{P_t^{H}})^{\frac{\partial}{\partial - 1}} Y_t^H}.
\]  

(A.23)

To facilitate the analysis, suppose for a moment that all retailers are allowed to adjust their prices every period \((\theta \to 0)\). In such a case, the optimality condition (A.23) reduces to

\[
\frac{\bar{P}_t^H}{P_t^H} = \mu \varphi^w_t,
\]

which is the standard result where each retailer set its price \(\bar{P}_t^H\) as a markup \(\mu \equiv \frac{\partial}{\partial - 1} > 1\) over the nominal marginal cost \(P_t^H \varphi^w_t\). If, in addition, prices were flexible, then the above expression reduces to \(\varphi^w_t = \frac{1}{\mu}\). Simultaneously, the marginal cost may be expressed in terms of the real wage rate over the marginal product of labor, namely \(\varphi^w_t = \frac{W_t}{P_t F_n;t}\), where \(F_n;t\) is the marginal product of labor. Combining these last two expressions leads to

\[
F_n;t = \mu \frac{W_t}{P_t}.
\]  

(A.24)

Equation (A.24) is the familiar condition in a monopolistically competitive environment where the marginal product of labor is equal to the real wage rate times the markup factor. Finally, substituting (A.24) into the optimality condition (A.15) yields

\[
\left( \frac{1}{\mu} \right) F_n;t = \frac{s C_t}{(1-\varsigma)(1-H_t)}.
\]

Hence, the term \(\frac{1}{\mu}\) introduces a wedge between the marginal product of labor and the marginal rate of substitution between leisure and aggregate consumption. Such a wedge may be captured by the term \(1 - \tau_{n,t} = \frac{s C_t}{(1-\varsigma)(1-H_t)} F_{n,t}\) in the associated prototype economy, as stated in the proposition. For the case of nominal price rigidities \((0 < \theta < 1)\), expression (A.23) still captures the idea of a wedge between the real wage and the marginal product of labor over time, which in turn affects the optimal intratemporal decision between leisure and consumption.

Now the idea is to show that the financial accelerator mechanism in the GGN model may be mapped into an investment wedge \(\frac{1}{1+\tau_{n,t}}\) in the associated prototype economy. First, let the tax on capital income be given by \(\tau_{k,t} = \frac{s_{x,t} \tilde{F}_{k,t}}{r_t}\). In such a case, the Euler equation (A.22) in the associated prototype economy may be written as
\[ \lambda_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{(1 + \tau_{x,t}) P_{k,t}} \left( r_{t+1} + \tilde{P}_{k,t+1} \right) \right\}. \] (A.25)

This intertemporal condition needs to be compared to its counterpart in the GGN economy. To this end, substitute (15) into (A.12). Next, using the definition of the gross real rate of return on capital (13) and expression (A.19) it may be obtained

\[ \lambda_t = \beta E_t \left\{ \left[ \frac{\lambda_{t+1}}{1 + \chi \left( \frac{B_{t+1}/P_{t+1}}{N_{t+1}} \right)} \right] Q_t \right\} \left[ \frac{P_{W,t+1}}{P_{t+1}} F_{k,t+1} + \frac{P_{I,t+1}}{P_{t+1}} \left( \frac{1}{\Phi' \left( \frac{1}{K_{t+1}} \right)} - \delta(u_{t+1}) \right) \right]. \] (A.26)

It may be readily shown that the intertemporal condition (A.26) from the GGN model may be recovered from (A.25) after substituting the relevant equations for \( r_t, \tilde{P}_{k,t} \) and the investment wedge \( 1/(1 + \tau_{x,t}) \) described in the proposition.

Finally, the government wedge is obtained by substituting the last two expressions of the proposition into the household’s budget constraint (21) and using the fact that profits \( \Pi_t \) are zero in the associated prototype economy.

**Appendix B**

The goal of this appendix is to illustrate that the properties of the investment wedge are also sensitive to the value of Tobin’s \( q \) elasticity when the whole period 1982:3 - 2005:2 is considered. Properties of the wedges are estimated using the results from Table 1. First, Table 2 shows the standard deviations relative to output and cross correlations for each of the four measured wedges during the period 1982:3 - 2005:2 using HP filtered data and assuming a Tobin’s \( q \) elasticity value of 0.5. Part A illustrates that the government wedge is highly volatile (relative to output), whereas the efficiency wedge is the least volatile among the four wedges. In addition, the efficiency, labor and investment wedges are positively correlated with output, both contemporaneously and for several leads and lags. In contrast, the government wedge is negatively correlated with output, both contemporaneously and for several leads and lags. Part B shows that the combination of efficiency and investment wedges, and labor and investment wedges are positively correlated, both contemporaneously and for several leads and lags.\(^{17}\)

\(^{17}\) As a reference, for the US postwar period Chari et al. (2006a) also report that the efficiency wedge
Table 3 presents the same exercise as in Table 2 but assuming a Tobin’s \( q \) elasticity value of infinity. Now the investment wedge is less volatile (relative to output) and negatively correlated with output, both contemporaneously and for several leads and lags. The cross correlations of the investment wedge with each of the remaining wedges are now opposite in sign to those reported in Table 2 with the exception of the efficiency wedge. However, all remaining results in Table 3 are essentially similar to those reported in Table 2. Thus what it may be concluded from Tables 2 and 3 is that the investment wedge is highly sensitive to the specification of Tobin’s \( q \) elasticity value, a result consistent with Figure 2.

Table 4 compares simulated movements in output due to a particular wedge to those of actual output for the period 1982:3 - 2005:2 using HP filtered data and assuming \( \eta = 0.5 \).

The first row in part A illustrates that movements in output due to the efficiency wedge alone have a standard deviation of 0.49 relative to output fluctuations in the data. Hence the efficiency wedge falls short from replicating output fluctuations. However, simulated output due to the efficiency wedge is positively correlated with output in the data, both contemporaneously and for several leads and lags. A positive correlation with output is also obtained for output fluctuations due to the labor wedge and the investment wedge, each acting in isolation. Their corresponding standard deviations relative to output are 1.23 and 0.45, respectively. Thus the labor wedge alone overestimates output fluctuations whereas the investment wedge alone underestimates them. Finally, in part B it is noticed that output due to the investment wedge is positively (negatively) correlated with output due to the efficiency and labor wedges (government wedge).

Table 5 replicates the numeric exercise of Table 4 assuming an infinite value for Tobin’s \( q \) elasticity. Now output fluctuations due to each of the four wedges are larger as the elasticity increases. However, cross correlations between simulated and actual output remain about the same for the labor wedge. For the investment wedge component, such a correlation is now negative. The sign for cross-correlations between investment and each of the other wedges is also reversed with the exception of the efficiency wedge.

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18 Chari et al. (2006a) find that the efficiency wedge in the US postwar period exhibits the largest standard deviation relative to output although its magnitude is less than one (0.73). As in Table 3 here, US output is positively correlated with simulated output arising from efficiency, labor and investment wedges, and negatively correlated with the government wedge.
The evidence shown in tables 4 and 5 suggests that the labor wedge alone is the most promising friction to explain output fluctuations in Korea for the period 1982:3 - 2005:2 when $\eta = 0.5$, followed by investment and efficiency wedges. As $\eta$ increases to infinity, the volatility in output due to the labor wedge is now about 2.73 times the volatility found in the data whereas the efficiency wedge alone continues to be a relatively appropriate friction to explain output movements. In contrast, the investment wedge in isolation is no longer a promising avenue to account for movements in Korean output. This is consistent with the results reported in Figures 3, 6 and 8. Finally, the government consumption wedge is not an appropriate friction in either case.

Appendix C

This appendix briefly discusses how variables are constructed as well as data sources.

Output is real GDP minus real taxes less subsidies on production. Investment includes real consumption of durable goods. As in Chari et al. (2006b), government consumption is defined as the sum of real government consumption plus real net exports of goods and services. All NIPA series are from the Korea National Statistical Office (KNSO). Output, investment and government consumption are divided by the population between 15 and 64 years as reported by the KNSO.

Employment is the number of persons aged 15 years and over (in both urban and rural areas) employed during the week of reference as reported by KNSO. Working hours are average effective working hours per week in the manufacturing sector from ILO LABORSTAT (until 2002), and average working hours in manufacturing from KNSO (2003:1 to 2005:2). Labor is defined in terms of working hours per capita (employment $\times$ working hours divided by population between 15 and 64 years).
Table 1
Parameters of Vector AR(1) Stochastic Process\textsuperscript{a}
Benchmark Model with Tobin’s q Elasticity = 0.5

Coefficient matrix on lagged states (P)

\[
\begin{bmatrix}
0.969 & -0.128 & -0.017 & 0.024 \\
0.897,1.040 & (-0.180,0.075) & (-0.040,0.006) & (0.006,0.042) \\
-0.117 & 0.628 & -0.081 & 0.072 \\
(-0.253,0.020) & (0.502,0.755) & (-0.137,0.025) & (0.033,0.110) \\
-0.028 & -0.221 & 0.924 & 0.058 \\
(-0.277,0.221) & (-0.404,0.038) & (0.835,1.012) & (-0.011,0.128) \\
-0.373 & -0.017 & 0.028 & 0.918 \\
(-0.687,0.059) & (-0.271,0.238) & (-0.075,0.131) & (0.823,1.014)
\end{bmatrix}
\]

Coefficient matrix on shocks (Q)

\[
\begin{bmatrix}
-0.022 & 0 & 0 & 0 \\
(-0.024,-0.019) \\
-0.021 & 0.047 & 0 & 0 \\
(-0.030,-0.011) & (0.039,0.055) \\
0.026 & 0.049 & 0.056 & 0 \\
(0.009,0.044) & (0.023,0.075) & (0.039,0.073) \\
0.014 & 0.084 & 0.003 & -0.109 \\
(-0.006,0.033) & (0.056,0.112) & (-0.025,0.031) & (-0.126,-0.091)
\end{bmatrix}
\]

Means of states = \[ [0.068 (0.047,0.090), -0.012 (-0.048,0.024), 0.116(0.067,0.164), -2.015 (-2.147, -1.883)] \]

\textsuperscript{a}The stochastic process is described by equation (25) in the main text. Parameters are estimated using maximum likelihood with data on output, labor, investment and government consumption. Numbers in parentheses are 90 percent confidence intervals from a bootstrapped distribution under 500 replications.
### Table 2

**PROPERTIES OF THE WEDGES WITH TOBIN’S Q ELASTICITY = 0.5, 1982:3-2005:2**

<table>
<thead>
<tr>
<th>Wedges</th>
<th>Standard Deviation Relative to Output</th>
<th>Cross Correlation of Wedge with Output at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.61</td>
<td>0.18</td>
</tr>
<tr>
<td>Labor</td>
<td>2.07</td>
<td>0.24</td>
</tr>
<tr>
<td>Investment</td>
<td>5.55</td>
<td>0.48</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>10.79</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wedges (X,Y)</th>
<th>Cross Correlation of X with Y at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency, Labor</td>
<td>0.12</td>
</tr>
<tr>
<td>Efficiency, Investment</td>
<td>0.11</td>
</tr>
<tr>
<td>Efficiency, Government Consumption</td>
<td>-0.19</td>
</tr>
<tr>
<td>Labor, Investment</td>
<td>0.30</td>
</tr>
<tr>
<td>Labor, Government Consumption</td>
<td>-0.34</td>
</tr>
<tr>
<td>Investment, Government Consumption</td>
<td>-0.46</td>
</tr>
</tbody>
</table>

*Statistics based on logged and HP-filtered series.

### Table 3

**PROPERTIES OF THE WEDGES WITH TOBIN’S Q ELASTICITY = INFINITY, 1982:3-2005:2**

<table>
<thead>
<tr>
<th>Wedges</th>
<th>Standard Deviation Relative to Output</th>
<th>Cross Correlation of Wedge with Output at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.61</td>
<td>0.20</td>
</tr>
<tr>
<td>Labor</td>
<td>2.07</td>
<td>0.24</td>
</tr>
<tr>
<td>Investment</td>
<td>1.66</td>
<td>-0.42</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>10.79</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wedges (X,Y)</th>
<th>Cross Correlation of X with Y at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency, Labor</td>
<td>0.13</td>
</tr>
<tr>
<td>Efficiency, Investment</td>
<td>-0.20</td>
</tr>
<tr>
<td>Efficiency, Government Consumption</td>
<td>-0.21</td>
</tr>
<tr>
<td>Labor, Investment</td>
<td>-0.17</td>
</tr>
<tr>
<td>Labor, Government Consumption</td>
<td>-0.34</td>
</tr>
<tr>
<td>Investment, Government Consumption</td>
<td>0.63</td>
</tr>
</tbody>
</table>

*Statistics based on logged and HP-filtered series.
### Table 4
PROPERTIES OF THE OUTPUT COMPONENTS WITH TOBIN’S Q ELASTICITY = 0.5, 1982:3-2005:2 *

<table>
<thead>
<tr>
<th>Output Components</th>
<th>Standard Deviation Relative to Output</th>
<th>Cross Correlation of Wedge with Output at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.49</td>
<td>0.19</td>
</tr>
<tr>
<td>Labor</td>
<td>1.23</td>
<td>0.45</td>
</tr>
<tr>
<td>Investment</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>1.06</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

### B. Cross Correlations

<table>
<thead>
<tr>
<th>Output Components (X,Y)</th>
<th>Cross Correlation of X with Y at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency, Labor</td>
<td>0.16</td>
</tr>
<tr>
<td>Efficiency, Investment</td>
<td>0.13</td>
</tr>
<tr>
<td>Efficiency, Government Consumption</td>
<td>-0.20</td>
</tr>
<tr>
<td>Labor, Investment</td>
<td>0.53</td>
</tr>
<tr>
<td>Labor, Government Consumption</td>
<td>-0.59</td>
</tr>
<tr>
<td>Investment, Government Consumption</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

*Statistics based on logged and HP-filtered series.

### Table 5
PROPERTIES OF THE OUTPUT COMPONENTS WITH TOBIN’S Q ELASTICITY = INFINITY, 1982:3-2005:2 *

<table>
<thead>
<tr>
<th>Output Components</th>
<th>Standard Deviation Relative to Output</th>
<th>Cross Correlation of Wedge with Output at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.86</td>
<td>0.08</td>
</tr>
<tr>
<td>Labor</td>
<td>2.73</td>
<td>0.45</td>
</tr>
<tr>
<td>Investment</td>
<td>1.13</td>
<td>-0.46</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>1.16</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

### B. Cross Correlations

<table>
<thead>
<tr>
<th>Output Components (X,Y)</th>
<th>Cross Correlation of X with Y at Lag k=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>Efficiency, Labor</td>
<td>0.03</td>
</tr>
<tr>
<td>Efficiency, Investment</td>
<td>-0.03</td>
</tr>
<tr>
<td>Efficiency, Government Consumption</td>
<td>-0.06</td>
</tr>
<tr>
<td>Labor, Investment</td>
<td>-0.44</td>
</tr>
<tr>
<td>Labor, Government Consumption</td>
<td>-0.61</td>
</tr>
<tr>
<td>Investment, Government Consumption</td>
<td>0.65</td>
</tr>
</tbody>
</table>

*Statistics based on logged and HP-filtered series.
Figure 1
Output and Measured Wedges for Tobin’s q Elasticity = 0.5
Benchmark Model
Figure 2
Output and Investment Wedges under Alternative Tobin q Elasticities
Benchmark model

Output

Elasticity = infinity

Elasticity = 3

Elasticity = 1

Elasticity = 1/2
Figure 3
Data and Predictions of Model with Just the Investment Wedge under Alternative Tobin's q Elasticities
Benchmark Model
Figure 4
Data and Predictions of Model with No Investment Wedge under Alternative Tobin's q Elasticities
Benchmark Model
Figure 5
Data and Predictions of Model with Just the Labor Wedge
Benchmark Model

Output


Labor


Investment


Data
Elasticity = infinity
Elasticity = 3
Elasticity = 1
Elasticity = 1/2
Figure 6
Data and Predictions of Model with Just the Investment Wedge under Alternative Tobin's q Elasticities
Model with Variable Capital Utilization

Output

Labor

Investment

Elasticity = infinity
Elasticity = 3
Elasticity = 1
Elasticity = 1/2
Figure 7
Data and Predictions of Model with No Investment Wedge under Alternative Tobin's q Elasticities
Model with Variable Capital Utilization

- Output
- Labor
- Investment

Legend:
- Data
- Elasticity = infinity
- Elasticity = 3
- Elasticity = 1
- Elasticity = 1/2
Figure 8
Data and Predictions of Model with Just the Investment Wedge under Alternative Tobin's q Elasticities
Benchmark Model with Measurement Errors

Output Elasticity = infinity
Output Elasticity = 3
Output Elasticity = 1
Output Elasticity = 1/2

Labor

Investment
Figure 9
Data and Predictions of Model with No Investment Wedge under Alternative Tobin's q Elasticities
Benchmark Model with Measurement Errors

Output Elasticity: $\infty$
- Elasticity: 3
- Elasticity: 1
- Elasticity: $\frac{1}{2}$

Labor

Investment

Legend:
- Blue: Data
- Green: Elasticity = $\infty$
- Red: Elasticity = 3
- Cyan: Elasticity = 1
- Pink: Elasticity = $\frac{1}{2}$

Data Points:
- 1997: 4
- 1998: 1
- 1998: 2
- 1998: 3
- 1998: 4
- 1999: 1
- 1999: 2
- 1999: 3
- 1999: 4
- 2000: 1
- 2000: 2

Y-axis:
- 1997: 4
- 1998: 1
- 1998: 2
- 1998: 3
- 1998: 4
- 1999: 1
- 1999: 2
- 1999: 3
- 1999: 4
- 2000: 1
- 2000: 2

X-axis:
- 1997: 4
- 1998: 1
- 1998: 2
- 1998: 3
- 1998: 4
- 1999: 1
- 1999: 2
- 1999: 3
- 1999: 4
- 2000: 1
- 2000: 2

Values:
- 80
- 85
- 90
- 95
- 100
- 105
- 110

- Labor
- Investment