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Using Seasonal Models to Forecast Short-Run Inflation in Mexico*

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Abstract: Since the adoption of inflation targeting, the seasonal appears to be the component that explains the major part of inflation’s total variation in Mexico. In this context, we study the performance of seasonal time series models to forecast short-run inflation. Using multi-horizon evaluation techniques, we examine the real-time forecasting performance of four well-known seasonal models using data on 16 indices of the Mexican Consumer Price Index (CPI), including headline and core inflation. These models consider both, deterministic and stochastic seasonality. After selecting the best forecasting model for each index, we apply and compare two methods that aggregate hierarchical time series, the bottom-up method and an optimal combination approach. The best forecasts are able to compete with those taken from surveys of experts.

Keywords: Aggregated forecasts, bottom-up forecasting, forecast combination, hierarchical time series, inflation targeting, multi-horizon evaluation, seasonal unit roots.

JEL Classification: C22, C52, C53, E37.

Resumen: Desde la adopción del esquema de objetivos de inflación, el componente estacional parece ser el que explica la mayor parte de la variación total de la inflación en México. En este contexto, se estudia el desempeño de modelos estacionales de series de tiempo para pronosticar la inflación de corto plazo. Mediante técnicas de evaluación para horizontes múltiples, se examina el desempeño en tiempo real de los pronósticos de cuatro modelos estacionales, utilizando datos de 16 índices del Índice Nacional de Precios al Consumidor (INPC), incluyendo la inflación general y la subyacente. Estos modelos consideran tanto estacionalidad determinística como estocástica. Una vez seleccionado el mejor modelo de pronóstico para cada uno de los índices, se aplican y comparan dos métodos de agregación de series de tiempo jerárquicas, el método bottom-up y un método de combinación óptima. Los mejores pronósticos logran competir con los reportados por las encuestas a especialistas.

Palabras Clave: Combinación de pronósticos, evaluación de horizontes múltiples, objetivo de inflación, pronósticos agregados, raíces unitarias estacionales, series de tiempo jerárquicas.

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1 Introduction

Models to forecast inflation have traditionally focused on the trend of inflation, since this component would typically explain most of the variation of the series, in line with Granger’s (1966) statement that most of the variation of economic time series is explained by the long-run trend. In particular, inflation’s trend component has usually been modeled as stochastic, by means of models that contain a unit root (e.g., Stock and Watson (2003)).

Nevertheless, under stable inflation, for example, that obtained under a credible inflation targeting regime, the trend loses importance as the dominant component (Stock and Watson, 2007). At present, this appears to be the case in Mexico. In particular, inflation in Mexico seems to have switched from a nonstationary to a stationary process around the end of 2000 or the beginning of 2001 (Chiquiar et al., 2007). As we show in the present paper, the component that seems to have replaced the trend as the dominant component is the seasonal.

In this context, with the purpose of finding models that can produce good forecasts of monthly inflation up to 12 months ahead, this document treats inflation as a seasonal series, and applies four time series models specifically designed to model and forecast them (Osborn, 2002). The models consider both, stochastic and deterministic seasonality, and are applied to 16 inflation series from the Mexican Consumer Price Index (CPI), including headline, and core inflation. The evaluation of each model is performed using out-of-sample forecasts, simultaneously taking into account all the forecast horizons.

Once the best model for each inflation series is determined, we face the problem of aggregating them in such a way that the resulting forecasts are consistent with the hierarchical order of the series (e.g., the headline price index has to equal a specific weighted average of the core and non-core price indices). To solve this problem, we compare two different methodologies, the commonly used bottom-up approach, and an optimal combination approach recently proposed by Hyndman et al. (2007), modified for the Mexican case. The forecasts produced with the latter not only satisfy the hierarchies, but in most cases have smaller mean squared forecast errors (MSFE) than both, the forecasts produced with the bottom-up approach, and the forecasts obtained with the best seasonal model for each series.

The document proceeds as follows. Section 2 presents the structure of the Mexican CPI and documents the recent changes that the seasonal component of Mexican inflation has experienced. Section 3 presents the four seasonal models and their evaluation, using the last 36 months of the sample to compare the forecasts with the actual values and to choose the best model for each of the 16 series. Section 4 introduces the discussion of whether to aggregate the forecasts of disaggregated variables or to forecast the aggregate variable.
of interest directly, and shows the evaluation of two alternative methods to aggregate the 16 resulting forecasts in a way that is consistent with the hierarchical order of the series. Finally, section 5 presents the conclusions.

2 The seasonal component in Mexican inflation data

2.1 Data

Every month Banco de México compiles 170,000 prices of specific goods and services which are then grouped into 315 items (“genéricos”, Banco de México, 2002b). Each item has a certain weight inside the Mexican CPI, with the weights determined depending on the importance that each good and service has in the consumption basket that represents the average Mexican consumer. The items are classified as part of groups, and then these groups are classified as elements of larger groups, leading to a hierarchical structure. In this paper we restrict our attention to the four higher levels of the hierarchy, which in the case of the Mexican CPI represents 16 series. This grouping is the most commonly used to monitor inflation in Mexico.

Table 1 presents the hierarchical structure of the Mexican CPI, while Figure 1 presents the time series of the 16 indices analyzed. The first disaggregation of the headline index is between core and non-core indices. The core index contains the less volatile items, and it is usually thought to respond to the aggregate economy and to monetary policy. For instance, prices in this index usually respond, with a lag, to domestic macroeconomic variables such as the interest rate, the exchange rate, and wages. The core index is disaggregated into merchandise and services indices, which are then disaggregated into food and other merchandise indices and into housing, education and other services indices, respectively. In the case of the non-core index, it is formed by the very volatile agricultural and livestock group and by the group with administered and regulated prices such as those from gasoline, electricity, telephone, and local transport, among others (Banco de México, 2002b). The non-core index responds mostly to international prices and to domestic non-market forces.

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1 Banco de México is Mexico’s central bank, with web page: http://www.banxico.org.mx

2 The information regarding the relative importance that each item has in the basket is obtained from a survey that the Instituto Nacional de Estadística y Geografía (INEGI) formulates to the Mexican households. This survey is known as the Encuesta Nacional de Ingresos y Gastos de los Hogares (ENIGH). For more information about the composition and the current weights of the Mexican CPI see Banco de México (2002a; 2002b).

3 The measure of core inflation that we use includes education. This is currently the definition of core inflation used by Banco de México (see Banco de México (2007)). However, before January 2008 Banco de México considered education part of non-core inflation. The historical series that we use is available at Banco de México’s website.
The former since most agricultural products and energy-related products are commodities, and the latter because an important part of administered and regulated prices are determined by the public sector. In particular, most energy prices under the administered group are determined by the federal government (e.g., gasoline), while a considerable part of the prices under the regulated group are regulated by sub-national governments (e.g., rights for water provision).4

The hierarchical structure of the time series that we analyze brings us to the discussion of whether to directly forecast an aggregate variable or to forecast disaggregate variables and then aggregate them. First, for the practical reason that the forecasts of the inflation of the 16 indices must be congruent with each other in the sense that they have to satisfy the hierarchies. Second, because of the possibility that certain aggregations methods may improve upon at least some of the individual forecasts (Espasa et al., 2002).

2.2 Importance of the seasonal component

One of the most accepted definitions of seasonality is the one from Hylleberg (1992, p.4): “Seasonality is the systematic, although not necessary regular, intra-year movement caused by the changes of the weather, the calendar, the timing of decision, directly or indirectly through the production and consumption decisions made by agents of the economy.” Since this is a systematic movement, it is very predictable, which makes the inclusion of seasonal factors relevant to forecast economic time series.

In Mexico, the seasonal part of inflation has recently gained importance in the sense that it now explains a larger amount of inflation’s total variation. This can be seen by analyzing the evolution of the spectrum of the inflation series. The spectrum represents the contribution of cycles of different frequencies to the variance of the series. It takes higher values in those frequencies whose cycles have a larger contribution to the variance of the observed series. Thus, series that are dominated by long run trends show a very particular form, with a larger portion of the density concentrated in lower frequencies, in what is known as the “typical spectral shape” of an economic variable, a name first used by Granger (1966). On the flip side, the spectrum of a monthly series with an important seasonal pattern will show jumps around the frequencies that correspond to 6, 5, 4, 3, 2, or 1 cycles per year, which correspond to cycles that are repeated every 2, 2.4, 3, 4, 6 and 12 months (Ghysels and Osborn, 2001). The estimated spectra for headline, core, and non-core inflation are presented in Figure 2 for two different samples: from April 1995 to April 2001, and from

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4With the main exception of telephone tariffs, which are determined privately. However, they are considered in the regulated group because a large part is regulated through a concession contract.
May 2001 to May 2007 (72 observations for each sample).\footnote{We choose to split the sample in April 2001 following the results in Chiquiar et al. (2007), who suggests that there may be a structural break in the persistence of inflation around that date. In addition, the same number of observations are incorporated in each sample for comparability.} Two changes are clear in the second sample: the drop of the peak at frequency zero, and the increase in the seasonal peaks.\footnote{Another interesting observation is the decrease of total variance, measured as the area below the curve of the spectral density, in the most recent sample of headline and core inflation. This does not seem to happen with the total variance of non-core inflation, except perhaps at very low frequencies.}

The changes in the spectra indicate that the seasonal could be the component that explains most of the total variation of inflation in recent times.\footnote{Other components, such as volatility, could have also increased its share of inflation’s total variation. Later on the paper it will be clear that this is the case for the inflation of fruits and vegetables.} To calculate the proportion of such variation that is explained by the seasonal component, we estimate a series of regressions for monthly inflation and its components with each regression having as regressors 12 seasonal dummy variables. The first dummy variable takes the value of 1 each January and the value of 0 the rest of the months, the second variable takes the value of 1 each February and the value of 0 the rest of the months and so on. For the first regression, the sample consists of the first five years of each series. For the second one, the first observation is dropped and a new observation is added at the end of the sample, so that the sample always contains 5 years of data (i.e., rolling windows are used for the estimation). In this context, the $R^2$ of each regression measures the percentage of inflation’s variation that is explained by the seasonal component. Figure 3 shows the resulting series of $R^2$s for each inflation component. The sample used in each case is constrained by data availability.

For headline inflation, the seasonal component goes from explaining less than 30% of the total variation of the series during the 80s and the first half of the 90s, to explain nearly 60% during the first five years of the 2000s, and even reaching the 70% mark in the last sample.

For core inflation, it is also the case that there is an important increase in the capacity of the seasonal component to explain the total variance, reaching almost 70% in the last five years. In the case of merchandise, the seasonal component goes from levels lower than 5%, to explain almost 60% of merchandise’s variation. The seasonal component of services inflation goes from explaining 10% during the early 90s, to explain nearly 70% of the total variation in the last sample. For the case of the components of merchandise inflation (available starting 1995) it is observed that the increase in the importance of seasonality is given primarily by the component of other merchandise, while for food inflation, the seasonal component explains between 20% and 65% of total variation, but without a clear pattern.\footnote{In Mexico, as in many other countries, processed food is classified as part of core inflation, while non-processed food is classified as part of non-core inflation. Nevertheless, as can be noticed in this paper, processed food’s prices behave somehow like non-processed food’s ones. Part of the reason is that some of}
services’ components, the importance of seasonality has remained relatively constant across samples. A special case is education, as it is almost completely determined by the seasonal component (with an $R^2$ around 90%); for some samples, it is possible to explain almost all the variation of education using only the seasonal component (e.g., for the sample from August 1995 to August 2000). The strong seasonal pattern in education occurs because most private schools change prices at the beginning of the academic year, in September (and a few also in January).

For non-core inflation, the seasonal component has been very important, explaining around 50% of the total variance of the series. In recent times, the seasonal reached the 70% mark. The seasonal component increases its importance for agricultural and livestock inflation during the last years of the 90s and the first years of the 2000s; however, lately seasonality has been decreasing in importance (nowadays explains around 30% of the variation of the series). This is explained by the behavior of the fruits and vegetables category, for which it is possible that the irregular component is dominating the variance of the series.\(^9\) On the other hand, it is noticeable the sharp increase in the importance of seasonality for the case of administered prices’ inflation. These days, the seasonal component explains more than 90% of the total variation of that series. This shows the regularity with which some prices and fees, such as electricity tariffs, have been revised. There is also a slightly increasing trend in the importance of the seasonal component for the inflation of regulated prices.

Although the importance of the seasonal component has been changing through time, we expect the percentage of total variation that is explained by the seasonal part to stabilize around its actual values, which seems reasonable, as long as inflation remains stationary.

### 2.3 Seasonal factors

The changes in the seasonal component of inflation go beyond the increments in its relative contribution to the total variance of the series. The seasonal factors have also changed. To see these changes, Figure 4 presents the seasonal factors for headline, core, and non-core inflation, estimated for 1996 and for each year between 2003 and 2007. Such factors are estimated using the TRAMO-SEATS methodology (Gómez and Maravall, 1996).\(^{10}\) It can the former move heavily with the price of commodities (e.g., the price of cereal can respond up to 90% to the price of wheat). Hence, it is debatable if processed food could be classified as part of non-core inflation.\(^9\) Other forces may be at play. For instance, changing weather patterns around the world may be changing the seasonality and the variability of agricultural products’ prices.

\(^{10}\)The conclusions that we draw in this subsection are robust to the use of X12-ARIMA to obtain the seasonal factors. However, some indices in the non-core part, such as fruits and vegetables, are more easily modeled using TRAMO-SEATS.
be seen that the seasonality of inflation has flattened, in the sense that now January and December have a less significant contribution to the series. Such flattening is clearer in the case of core inflation. Part of the shift in the seasonal factors occurred due to the change in the weights used to calculate the CPI in June 2002 (Banco de México, 2002b). The change was made to update the estimates of the expenditure shares that families assign to each good and service that are part of the index, and also to change the basket of goods and services considered. Another important factor that could also have influenced the shift in the seasonal factors is the transition from high to low inflation.

The seasonal factors have been stable in recent times, as can be seen in Figure 5, which shows headline, core, and non-core inflation per-year from 2003 to 2007 plus the average from 1998 to 2000. Several characteristics can be noticed by comparing the average from 1998 to 2000 with the rest of the years displayed: First, the reduction in the importance of the long-run trend; second, the change in the relative contribution of each month; and, finally, that inflation has followed a seasonal pattern quite stable since, at least, 2003. The seasonal factors, which appear stable, show that core inflation is higher during the first months of the year and in September. The first months of the year are high probably because most of the re-pricing in goods and services (in particular housing and education) is done at the beginning of the year, and September is high because of the mentioned changes in the prices of education at the beginning of the academic year. Non-core inflation is higher during the second half of the year and is notably low during May. The very low levels in May and the very high levels in November correspond to the beginning and the end, respectively, of the subsidy to electricity tariffs for the warm season. August and September are relatively high in part as the result of increments in the prices of fruits and vegetables (possibly reflecting the hurricane season). January is relatively high because most of the changes in prices of regulated goods and services have occurred at the beginning of the year in our sample (e.g., changes to the prices of public transportation).

The seasonal factors are likely to remain stable, at least until the weights used to calculate the CPI are changed again, and provided inflation remains low. With respect to the latter, Gagnon (2007), using data for Mexico, shows that when inflation is below 10-15% per year the frequency of price changes is only mildly correlated with the level of inflation, in particular for goods, but that this frequency is strongly correlated with inflation when the annual inflation rate is high (above 10-15%). Hence, under low inflation pricing decisions are more in line with time-dependent price setting models, which is congruent with stable seasonal

\[11\] 36 goods were added, among them bottled water, flour tortillas, and internet services, while some were dropped, for example train fees.

\[12\] A change in the weights or in the basket of goods and services used to calculate the CPI, most probably would change the seasonal factors, although it could occur that the factors remain the same.
patterns of price setting. In contrast, under high inflation pricing decisions display strong state-dependence, making the timing of price changes an endogenous decision, distorting the seasonality of price changes.

In this context, models specifically designed to model and forecast seasonal time series may be successfully applied to forecast Mexican inflation and its components in the short-run.

3 Models for forecasting seasonal time series

In this section we analyze two different classes of seasonal models. The first class corresponds to models that assume seasonal unit roots. This kind of models suggests that, in order to induce stationarity, it is necessary to seasonally differentiate the series. The second class consists of models that assume deterministic seasonality. In this case the seasonality can be taken into account using seasonal dummy variables. In total, we evaluate four models, two of each class. These models have been suggested in the literature as good models to forecast time series with strong seasonal components (Osborn, 2002).\footnote{When a time series shows a strong seasonal component that changes slowly across time, periodic models or models with time-changing parameters are the most recommended for forecasting (Franses, 2007). Nevertheless, such models require long time series –more than three decades of seasonal data (Franses, 1996) and, hence, are not considered in this paper.}

While it is possible to perform seasonal unit root tests (Hylleberg et al. (1990), and Rodrigues and Osborn (1999), among others), which may be helpful to select the best models to forecasts a series, these tests, in general, require a lot of information and do not have much power.\footnote{See Diebold and Kilian (2000) on the use of unit root pre-testing to select forecasting models.} The problem is aggravated in small samples, such as the ones used here. Nevertheless, if seasonality is stochastic, this has long-term consequences and a model that uses this information should make more accurate forecasts than one that ignores it, since the presence of seasonal unit roots makes the intra-year movements unpredictable in the long-run (Ghysels et al., 2005). The strategy we follow is to let the out-of-sample forecast evaluation tell us, for each series, which assumption about the stochastic nature of seasonality renders better forecasts.

3.1 Seasonal models

3.1.1 Models with seasonal unit roots

Model 1:

\[ \phi (L) \Delta_{12} y_t = \mu + \varepsilon_t, \]  

\[(1)\]
where $\pi_t$ is the inflation at time $t$, $\phi(L) = 1 - \phi_1 L - \phi_2 L - ... - \phi_p L$ is an autoregressive polynomial, with all its roots outside the unit circle (this assumption is made for all the models below), $L\pi_t = \pi_{t-1}$, $\Delta_{12}\pi_t = \pi_t - \pi_{t-12}$, $\mu$ is a constant, and $\varepsilon_t$ is a white noise process (also in the models below). This model assumes that the series $\pi_t$ has a unit root at the zero frequency and a seasonal unit root every month. It is possible to view this model as 12 different processes, all integrated of order one, where each one represents a month of the year.

Model 2:

$$\phi(L) \Delta_1 \Delta_{12} \pi_t = \varepsilon_t.$$  (2)

This model is similar to model 1, in the sense that it assumes a seasonal unit root every month. However, model 2 assumes two unit roots at the zero frequency. As can be seen, by applying the double-difference operator $\Delta_1 \Delta_{12}$ the model includes two conventional first differences. Leaving aside the seasonal unit-roots, this implies that the series is integrated of order two. This model is known as the “Airline Model” because it was successfully used to model the demand for passengers of air transports, which shows a very strong seasonality (Box et al., 1994).  

### 3.1.2 Models with deterministic seasonality

Model 3:

$$\phi(L) \Delta_1 \pi_t = \alpha_1 D_{1t} + ... + \alpha_{12} D_{12t} + \varepsilon_t,$$  (3)

where $D_{it}$ are seasonal dummy variables which take the value of one when the observation $t$ falls in the month $i$ and zero otherwise. This model assumes that the seasonality is deterministic, but that there is a long-run stochastic trend. This model allows the mean to vary with the month, in a deterministic manner.

Model 4:

$$\phi(L) \pi_t = \alpha_1 D_{1t} + ... + \alpha_{12} D_{12t} + \beta t + \varepsilon_t.$$  (4)

It is similar to model 3, but it assumes a long-run deterministic trend instead of a stochastic one. Notice that stochastic processes with no trend can be accomodated by this model (with $\beta = 0$).

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15 Notice that model 1 is an “Airline Model” applied to the price index.
### 3.2 Selection of the best seasonal model

We estimate the four models for each of the 16 series using Ordinary Least Squares (OLS) in a recursive manner. The sample starts in June 2002. The first sample ends in January 2005. The order of the lag polynomials is selected according to the Bayesian Information Criterion (BIC: Schwarz (1978)) each time that a model is estimated. The selection exercise uses the months from February 2005 to December 2007 to evaluate the out-of-sample forecasts. For the first sample, forecasts for 1 to 12 steps-ahead are computed (i.e., with information up to January 2005, forecasts from February 2005 to January 2006 are computed). For the second sample, which ends in February 2005, another 12 forecasts are computed, this time from March 2005 to February 2006. The exercise is repeated until December 2006 is incorporated in the last sample, and forecasts from January 2007 to December 2007 are calculated. At the end of the exercise, we have, for each of the 16 series, 24 multi-horizon forecasts, where each multi-horizon forecast contains forecasts for one- to twelve-steps-ahead.

Following Capistrán (2006), we consider that one model is preferred to another if its expected loss, defined over multiple-horizons, is smaller. For the evaluation, we use the Squared Error Loss (SEL) function, defined as:

\[
L_{t+1:t+12} = e'_{t+1:t+12} \Delta e_{t+1:t+12},
\]

with \( e_{t+1:t+12} = [e_{t+1}, \ldots, e_{t+12}]' \), \( e_{t+12} = \pi_{t+h} - f_{t+h:t} \), where \( \pi_{t+h} \) is the variable of interest, in this case inflation, at period \( t+h \), and \( f_{t+h:t} \) is the forecast of it made with information up to period \( t \). \( \Delta \) is an \( 12 \times 12 \) diagonal weighting matrix. We use the identity matrix for \( \Delta \), hence:

\[
L_{t+1:t+12,t} = e'_{t+1:t+12,t} I_{12} e_{t+1:t+12,t} = \sum_{i=1}^{12} (\pi_{t+i} - f_{t+i,t})^2.
\]

We calculate, for each series and model, loss sequences with 24 elements each. Then we take the average and the square root of it to calculate a multi-horizon Root Mean Squared Forecast Error (RMSFE). Notice that by using the identity matrix, equal weights are attached to each horizon and \( E[L_{t+1:t+h,t}] \) is the trace of the Mean Squared Error (MSE) matrix. In this case, the SEL can be related to the case of minimizing the MSE for each horizon (Capistrán, 2006). In using this particular loss function, we are assuming separability across horizons, that all horizons are equally relevant, and that the costs of the errors are symmetric for each horizon. But if the user of the forecasts cares more about some horizons, the loss function (5) can accommodate this by changing \( \Delta \) accordingly. For instance, if the

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\(^{16}\)The sample starts in June 2002 because seasonal factors are stable since that date and inflation appears to be stationary, as documented above.
first three horizons are considered more important, then the first three entries in the diagonal of \( \Delta \) should be higher than the other entries in the diagonal.

Table 2 presents the RMSFE for each model and index. The best models are those with the smaller RMSFE. Among the models that assume seasonal unit roots, model 1 turns out to be much better than model 2, as it shows a better performance for all the series. Among the models with deterministic seasonality, model 4 is better than model 3 for almost all the series, except for the cases of merchandise and food. Overall, models with deterministic seasonality seem to have a better performance at forecasting headline inflation and its components. Only for the cases of services and education, model 1, with seasonal unit roots, turns out to have a smaller RMSFE.

The result that models with deterministic seasonality perform better than models with seasonal unit roots is evidence in favor of the hypothesis that the seasonal component of inflation is probably deterministic for most series, at least in the samples used here. In exercises not reported, we noticed that there has been a tendency over time for the models with stochastic seasonality to be outperformed by models with deterministic seasonality.\(^\text{17}\) This indicates that inflation and some of its components may have had stochastic unit roots that have been vanishing. This would be congruent with the inflation targeting framework established by Banco de México since 2001, and would also be in line with the results of Chiquiar et al. (2007), that the zero frequency unit root seems to have disappeared from headline and core inflation.

### 4 Aggregation of seasonal models’ forecasts

When every inflation series is forecasted in a separate way, as we did in the previous section, the forecasts do not respect the hierarchies among the series. This means, for example, that the forecast for headline inflation that can be formed aggregating the best forecast for core inflation and the best forecast for non-core inflation, will be different than the forecast for headline inflation generated using the best seasonal model as selected in section 3.

In order to solve this practical problem, we evaluate two aggregation methods: the commonly used Bottom-Up method, and a method recently proposed by Hyndman, et al. (2007), which they call “optimal combination forecast for hierarchical time series” (we call it “HAA” for the rest of the paper). Both methods solve the problem. The first, although widely used, throws away information because only uses the forecasts from the lower level (9 in our case). The second uses all the forecasts (16), and it does the aggregation not only satisfying the

\(^{17}\)For instance, if we use data before 2002 the models that assume seasonal unit roots have a relatively better performance.
hierarchies, but also combining the forecasts in a way that, in principle, could return forecasts with the smallest possible variance among the aggregations that satisfy the hierarchies. Regardless of the method of aggregation, the forecasts to be aggregated are those selected in section 3.

A practical point that is important is that the expenditure weights used by Banco de México to construct the price indices apply directly to them and not to inflation. Therefore, although we forecast inflation, to aggregate we use the implied forecasts for the indices and then, once the aggregation is done, we transform the indices back to inflation.

4.1 Aggregation methods for hierarchical time series

Following the notation of Hyndman et al. (2007), and focusing in the case of Mexican CPI, the completely aggregated series, headline CPI, is assigned level 0. Level 1 is the first level of disaggregation which, in our case, includes both, the core and the non-core indices. Level 2 consists of the CPI for merchandise and for services (inside the core index), and for agricultural and livestock products and administered and regulated goods and services (inside the non-core index). Finally, level 3 contains the rest of the series: food, other merchandise, housing, education, other services, fruits and vegetables, livestock, administered, and regulated, for a total of 9 series at level 3.

Let’s call $m_i$ the total number of series that are in level $i$, $i = 0, 1, 2, 3$. Overall, the total number of series is $m = m_0 + m_1 + m_2 + m_3 = 16$. Defining everything in matrix and vector expressions, $P_{i,t}$ will be the vector of all the observations of level $i$ at time $t$, and $P_t = [P_{0,t}, P_{1,t}, P_{2,t}, P_{3,t}]’$. Note that:

$$P_t = SP_{3,t}, \quad (7)$$

where $S$ is a weighting matrix of order $m \times m_3$. Hyndman et al. (2007) use an $S$ matrix that only contains zeros and ones, whereas we have to incorporate the weights that reflect the composition of the Mexican CPI (as discussed above and reflected in Table 1). In our
case, the matrix $S$ and the vector $P_t$ have the following structure:

$$ S = \begin{bmatrix} 0.15 & 0.22 & 0.18 & 0.05 & 0.15 & 0.03 & 0.05 & 0.08 & 0.09 \\ 0.20 & 0.30 & 0.23 & 0.07 & 0.20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.13 & 0.19 & 0.31 & 0.36 \\ 0.40 & 0.60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.47 & 0.14 & 0.39 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.41 & 0.59 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.45 & 0.55 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} $$

$$ P_t = \begin{bmatrix} P_{H,t} \\ P_{S,t} \\ P_{N,t} \\ P_{SM,t} \\ P_{SS,t} \\ P_{NAg,t} \\ P_{NAc,t} \\ P_{SMA,t} \\ P_{SMO,t} \\ P_{SSV,t} \\ P_{SSE,t} \\ P_{SSO,t} \\ P_{NAgF,t} \\ P_{NAgC,t} \\ P_{NAcA,t} \\ P_{NAcC,t} \end{bmatrix}, $$

where $P_H$ is headline CPI, $P_S$ is the core CPI, $P_N$ is the non-core CPI, $P_{SM}$ is the merchandise index, $P_{SS}$ is the services index, $P_{NAg}$ is the agricultural and livestock index, $P_{NAc}$ is the administered and regulated index, $P_{SMA}$ is the food index, $P_{SMO}$ is the other merchandise index, $P_{SSV}$ is the housing index, $P_{SSE}$ is the education index, $P_{SSO}$ is the other services index, $P_{NAgF}$ is the fruits and vegetables index, $P_{NAgC}$ is the livestock index, $P_{NAcA}$ is the administered index, and $P_{NAcC}$ is the regulated index.

Taking the matrix $S$ and the best seasonal multi-horizon forecasts computed in the previous section, which will be denoted by $\hat{P}$, the aggregated forecasts are computed in the following way:

$$ \tilde{P} = SQ\hat{P}, $$

where $\tilde{P}$ and $\hat{P}$ are matrices of $16 \times 12$, whose elements are the forecasts of each of the 16 series for the 12 horizons; $S$ is the weighting matrix; and $Q$ is a matrix of order $m_3 \times m$, that varies according to the combination method.\textsuperscript{18} The purpose of $Q$ is to extract and combine the relevant elements of the baseline forecasts, $\hat{P}$, to get combined forecast of the lower level (level 3 in our case), which are later weighted by $S$ to aggregate the different levels, $\tilde{P}$. These forecasts are linear combinations of the baseline forecasts, which satisfy the hierarchies. In

\textsuperscript{18}The matrix $Q$ is equivalent to the matrix $P$ in Hyndman, et al. (2007).
addition, Hyndman et al. (2007), show that the resulting forecasts are unbiased as long as the baseline forecasts are unbiased and

\[ \text{SQS} = \text{S}. \]  \hspace{1cm} (9)

4.1.1 Bottom-Up method

This method uses only the forecasts from the most disaggregated series, the series in level 3, and then aggregates them using the corresponding weights, in matrix \( S \), to obtain forecasts for the rest of the levels of the hierarchy, levels 0, 1 and 2.

In this case, matrix \( Q \) takes the form:

\[
Q = \begin{bmatrix} 0_{m_3 \times (m-m_3)} & I_{m_3} \end{bmatrix},
\]

where \( 0 \) is a null matrix of order \( 9 \times 7 \) and \( I_{m_3} \) is an identity matrix of order \( 9 \times 9 \). Clearly, \( \text{SQS} = \text{S} \) which by construction returns unbiased forecasts if the forecasts for level 3 are unbiased.

4.1.2 HAA method

This method uses the forecasts for all the levels. In this case, the matrix \( Q \) takes the form:

\[
Q = (S'S)^{-1} S'.
\]

Hyndman et al. (2007) show that this method generates forecasts that satisfy the hierarchy, that are unbiased if the baseline forecasts are unbiased (since \( \text{SQS} = \text{S} \) and that, under certain assumptions, have the minimum variance among the possible combinations that comply with the above two points. The intuition behind the HAA method is that there are forecasts of the lower levels of the hierarchy implicit in the more aggregated levels (as one would find out using a top-down approach). But these implicit forecasts are constructed conditional on an information set that is probably broader than the set used to construct the forecasts for the lower levels. By combining all the possible forecasts to form better forecasts of the lower levels (9 series in our case), one is implicitly combining the conditional sets used to form all the forecasts (16 in our application). The matrix \( Q \) contains the combination weights, whereas the matrix \( S \) contains the aggregation weights. Notice that the weights in \( Q \) do not depend on the data, and that condition (9) implies that the combination of all the possible forecasts implicit for each particular series at the bottom of the hierarchy is convex (i.e., the weights add to one).
4.2 Evaluation of aggregation methods

In this subsection of the paper we first compare the predictive ability of the two aggregation methods described above, and then compare the predictive ability of the aggregated forecasts versus that of the best individual models.

4.2.1 Bottom-up vs. HAA

In order to compare the performance of both aggregation methods, for each series we use the following hypothesis:

\[ H_0: E[L_{t+1:t+12,t}^{BU} - L_{t+1:t+12,t}^{HAA}] = 0 \]
\[ H_1: E[L_{t+1:t+12,t}^{BU} - L_{t+1:t+12,t}^{HAA}] \neq 0, \]

where the BU superscript indicates that forecasts were aggregated using the bottom-up approach, whereas the HAA superscript indicates that the HAA combination method was used. The null hypothesis implies equal predictive ability and the alternative hypothesis implies that one forecast has smaller (multi-horizon) MSFE, in population. We form the test statistic as in Capistrán (2006), who proposes Diebold-Mariano-West type of tests (Diebold and Mariano, 1995; West 1996) applied to this type of multivariate loss functions. The test statistic is then compared to a standard normal distribution as the previous authors.

The results are presented in Table 3. The column labeled BU vs HAA shows the sample mean of the loss differential between BU and HAA, which forms the basis for the Diebold-Mariano-West test. The methods seem to have the same predictive power for 10 out of the 16 series. The HAA method is better than the Bottom-Up method for 3 series: Services, education, and other services. The Bottom-up method outperforms HAA for 3 series: Other merchandise; livestock, and regulated. Hence the methods seem to have similar prediction ability. However, notice that, although not statistically significant in some cases, HAA has a smaller average loss for 11 of the 16 series (as seen by the positive sign of the sample mean of the loss differential) and, hence, may be slightly preferred to the Bottom-up approach. In particular, HAA seems better for almost every series that does not belong to the lower level of the hierarchy, the exception been administered and regulated. This last result is consistent with the results of Hyndman et al. (2007).
4.2.2 Forecasting individual series vs. forecasting the aggregates by disaggregates

An interesting empirical issue that can be investigated with the forecasts produced in this paper is that of forecasting an aggregate directly versus forecasting the disaggregated series and then aggregate (e.g., Hubrich (2005)). In order to do this, we repeat the evaluation exercise but this time comparing the performance of the HAA method against the best seasonal model (without aggregation, as derived in section 3) for each of the 16 series. The results are presented in the column labeled Seasonal vs. HAA in Table 3. The number shown for each series is the average multi-horizon loss differential between the forecasts resulting from the best individual forecasts and the HAA method. The HAA method turns out to have better predictive ability for 6 series, whereas the individual models do for 3 series. All series for which the individual models appear to perform better belong to the lower level in the hierarchy. In addition, in every series that does not belong to level 3 of the hierarchy the HAA method is better (although the difference is not always statistically significant). Hence, our results indicate that, in this case, forecasting the components and then aggregating seems to yield better forecasts than directly forecasting the aggregates.

5 Conclusions

The forecasting ability of time series models has been widely documented (e.g., Granger and Newbold (1986)) and, in general, this class of models constitutes a good way to summarize what the past of a series can inform about its future. In this paper we have used a particular subset of time series models, those that specifically model the seasonal component, to forecast short-run inflation in Mexico. The four seasonal models that we consider have different assumptions about the stochastic properties of the trend and of the seasonal components of the series. We choose the best model for each of 16 series of inflation using a multi-horizon loss function, and then aggregate the resulting forecasts so that they satisfy the hierarchies among them.

The best individual models, in terms of out-of-sample RMSFE, seem to be those assuming the seasonal and the trend (when it is significant) as deterministic. The best forecasts are obtained by combining the individual models so as to make better use of the information contained in all of them while satisfying the hierarchies.

The resulting forecasts can be obtained in real-time in an automatic way, in the sense that once the information about inflation for a new month arrives, it takes seconds to re-estimate the models, select the best for each series, and combine the best individual forecasts.
Therefore, the forecasts obtained with the methods used here can be used as a good starting point in the recurrent process of forecasting inflation up to one year hence. In practice, once the automatic forecasts are obtained, one can adjust them in order to incorporate information that it is not contained in the past of the series (e.g., adjust them for a known coming change on administered prices), or combine them with forecasts from other sources.

Our best forecasts for headline inflation fare surprisingly well when compared against the consensus of the forecasters that answer the monthly Survey of Specialists in Economics from the Private Sector, maintained by Banco de México (EEBM). Figure 6 plots the forecasts from the consensus, the forecasts from the HAA method, and the actual value of inflation. The forecasts correspond to horizons one and twelve. It can be seen that the forecasts are very close to each other. A formal predictive ability test of the Diebold-Mariano-West type using the multi-horizon loss function produces a p-value of 0.49, indicating that there is not enough evidence to reject the null hypothesis of equal predictive ability. Considering that the forecasts from surveys of experts appear to have a better performance than other type of inflation forecasts (see Ang et al. (2007) for the United States, and Capistrán and López-Moctezuma (2008) for Mexico), this result shows that the forecasts produced with seasonal models can be used in a reliable manner as an automatic first forecast of short-run inflation, with the additional advantage that the methods used here can be applied to forecast any level of the hierarchy of the inflation series, whereas the forecasts obtained from surveys correspond typically to, at the most, the first two levels of the hierarchy.

Several ways to improve the forecasts presented here suggest themselves. In particular, the aggregation method suggests that obtaining better individual models, possibly incorporating predictor variables to some or all of them, may be a parsimonious way to introduce additional information. Furthermore, the combination weights implicit in the HAA aggregation method do not depend on the data and are restricted to add to unity, which implies that there may be room to improve upon them. Future research should look at these issues.

References


<table>
<thead>
<tr>
<th>Category</th>
<th>Weight</th>
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<tbody>
<tr>
<td>Headline</td>
<td>100%</td>
</tr>
<tr>
<td>Core</td>
<td>75%</td>
</tr>
<tr>
<td>Merchandise</td>
<td>37%</td>
</tr>
<tr>
<td>Food</td>
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</tr>
<tr>
<td>Other Merchandise</td>
<td>22%</td>
</tr>
<tr>
<td>Services</td>
<td>38%</td>
</tr>
<tr>
<td>Housing</td>
<td>18%</td>
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<tr>
<td>Education</td>
<td>5%</td>
</tr>
<tr>
<td>Other Services</td>
<td>15%</td>
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<tr>
<td>Non-Core</td>
<td>25%</td>
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<tr>
<td>Agricultural and Livestock</td>
<td>8%</td>
</tr>
<tr>
<td>Fruits and Vegetables</td>
<td>3%</td>
</tr>
<tr>
<td>Livestock</td>
<td>5%</td>
</tr>
<tr>
<td>Administered and Regulated</td>
<td>17%</td>
</tr>
<tr>
<td>Administered</td>
<td>8%</td>
</tr>
<tr>
<td>Regulated</td>
<td>9%</td>
</tr>
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</table>

Note: Weight represents the percentage weight that each index has on headline.

Source: Banco de México
Table 2: RMSFEs to determine the best seasonal model.

<table>
<thead>
<tr>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Winner</th>
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<tbody>
<tr>
<td>Headline</td>
<td>1.013</td>
<td>2.554</td>
<td>1.121</td>
<td>0.725</td>
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<tr>
<td>Core</td>
<td>0.364</td>
<td>0.737</td>
<td>0.366</td>
<td>0.341</td>
<td>4</td>
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<tr>
<td>Merchandise</td>
<td>0.484</td>
<td>1.190</td>
<td>0.466</td>
<td>0.496</td>
<td>3</td>
</tr>
<tr>
<td>Food</td>
<td>1.023</td>
<td>2.708</td>
<td>0.964</td>
<td>1.030</td>
<td>3</td>
</tr>
<tr>
<td>Other Merchandise</td>
<td>0.318</td>
<td>1.614</td>
<td>0.314</td>
<td>0.302</td>
<td>4</td>
</tr>
<tr>
<td>Services</td>
<td>0.511</td>
<td>1.800</td>
<td>0.557</td>
<td>0.521</td>
<td>1</td>
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<tr>
<td>Housing</td>
<td>0.648</td>
<td>2.235</td>
<td>0.588</td>
<td>0.544</td>
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</tr>
<tr>
<td>Education</td>
<td>0.601</td>
<td>6.194</td>
<td>1.919</td>
<td>1.477</td>
<td>1</td>
</tr>
<tr>
<td>Other Services</td>
<td>0.899</td>
<td>2.176</td>
<td>0.928</td>
<td>0.860</td>
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<tr>
<td>Non-Core</td>
<td>3.892</td>
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<td>4.342</td>
<td>2.694</td>
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</tr>
<tr>
<td>Agricultural and Livestock</td>
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<td>12.730</td>
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<tr>
<td>Fruits and Vegetables</td>
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<td>31.934</td>
<td>21.134</td>
<td>4</td>
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<tr>
<td>Livestock</td>
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<td>6.597</td>
<td>3.824</td>
<td>3.245</td>
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<tr>
<td>Administered and Regulated</td>
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<td>2.151</td>
<td>1.594</td>
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<tr>
<td>Administered</td>
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<td>15.455</td>
<td>3.882</td>
<td>2.897</td>
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</tr>
<tr>
<td>Regulated</td>
<td>0.809</td>
<td>2.410</td>
<td>1.078</td>
<td>0.799</td>
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</table>

Note: The RMSFEs are calculated with out-of-sample recursive forecasts from February 2005 to December 2007. The winner is the model with the smallest RMSFE.

Source: Own calculations with data from Banco de México.
<table>
<thead>
<tr>
<th>Category</th>
<th>BU vs. HAA</th>
<th>Seasonal vs. HAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headline</td>
<td>0.010</td>
<td>0.012</td>
</tr>
<tr>
<td>Core</td>
<td>0.002</td>
<td>0.008</td>
</tr>
<tr>
<td>Merchandise</td>
<td>0.004</td>
<td>0.025**</td>
</tr>
<tr>
<td>Food</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>Other Merchandise</td>
<td>-0.011**</td>
<td>-0.011**</td>
</tr>
<tr>
<td>Services</td>
<td>0.011***</td>
<td>0.044***</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>Education</td>
<td>0.005***</td>
<td>0.005***</td>
</tr>
<tr>
<td>Other Services</td>
<td>0.025***</td>
<td>0.025***</td>
</tr>
<tr>
<td>Non-Core</td>
<td>0.182</td>
<td>0.157</td>
</tr>
<tr>
<td>Agricultural and Livestock</td>
<td>0.427</td>
<td>2.095**</td>
</tr>
<tr>
<td>Fruits and Vegetables</td>
<td>1.184</td>
<td>1.184</td>
</tr>
<tr>
<td>Livestock</td>
<td>-0.568**</td>
<td>-0.568**</td>
</tr>
<tr>
<td>Administered and Regulated</td>
<td>-0.052</td>
<td>0.100**</td>
</tr>
<tr>
<td>Administered</td>
<td>-0.065</td>
<td>-0.065</td>
</tr>
<tr>
<td>Regulated</td>
<td>-0.050*</td>
<td>-0.050*</td>
</tr>
</tbody>
</table>

Note: Each test uses out-of-sample recursive forecasts from February 2005 to December 2007. *, **, and *** denote statistical significance at 10, 5 and 1 percent, respectively. The reported number is the average loss differential.

Source: Own calculations with data from Banco de México.
Figure 1: Inflation.

- **Headline**
- **Core**
- **Non-Core**
- **Merchandise**
- **Food**
- **Other Merchandise**
- **Services**
- **Housing**
Source: Banco de México.
Figure 2: Spectral densities.

Panel A. Headline Inflation

Panel B. Core Inflation

Panel C. Non-Core Inflation

Note: Densities estimated with Bartlett windows.
Source: Own calculations with data from Banco de México.
Figure 3: Goodness of fit of regressions that only include 12 seasonal dichotomic variables.
Note: $R^2$s of 5-year window rolling regressions of inflation on seasonal dummies.
Source: Own calculations with data from Banco de México.
Figure 4: Seasonal factors.

Panel A. Headline Inflation

Panel B. Core Inflation

Panel C. Non-Core Inflation

Note: Seasonal factors computed with Tramo-Seats.
Source: Own calculations with data from Banco de México.
Figure 5: Inflation dynamics.

Panel A. Headline Inflation

Panel B. Core Inflation

Panel C. Non-Core Inflation

Source: Banco de México.
Figure 6: Comparison of forecast combinations: Headline inflation.

Panel A. $h = 1$

Panel B. $h = 12$