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November 2014

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The informal sector in contemporary models of the aggregate economy*

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Abstract: I review a contemporary branch of the informal sector literature that focus on understanding the way firm behavior is affected by the presence of informality and how such distortions have an impact on aggregate variables. The authors in this group all make use of dynamic general equilibrium (DGE) models. I focus on models with heterogeneous firms and a cost of informality that is increasing with firm size: reducing informality entails a tradeoff because there are some distortions associated with the formal sector and some others with the informal. Quantitative evaluations of this tradeoff using these models show that, in general, reducing informality brings gains. In conclusion, substantial progress has been made in understanding informality and its consequences through the use of DGE models with heterogeneous firms. More research is needed to understand how informality affects the economy when other sources of heterogeneity are considered.

Keywords: informality, literature survey, dynamic general equilibrium, heterogeneous firms, distortions, productivity

JEL Classification: E26, O17, O40

Resumen: En este artículo reviso la literatura contemporánea del sector informal que se enfoca en entender la forma en que el comportamiento de las empresas es afectado por la presencia del sector informal y cómo esto impacta variables agregadas. Esta literatura hace uso de modelos de equilibrio general dinámico que incluyen firmas heterogéneas y un costo de la informalidad que es creciente con el tamaño de la empresa: el reducir la informalidad conlleva una disyuntiva porque existen algunas distorsiones asociadas con el sector formal y otras asociadas con el informal. La evaluación cuantitativa de esta disyuntiva a través de estos modelos muestra que, en general, reducir la informalidad trae ganancias. En conclusión, el uso de estos modelos ha permitido progresar en el entendimiento del sector informal y sus consecuencias. Más investigación es necesaria que estudie cómo la informalidad afecta la economía cuando se consideran otras fuentes de heterogeneidad.

Palabras Clave: informalidad, revisión de literatura, equilibrio general dinámico, empresas heterogéneas, distorsiones, productividad

*I would like to thank Stephen McKnight for comments and suggestions. Roberto Gomez-Cram has provided excellent research assistance. This paper will be a chapter of the forthcoming book "Contemporary Topics in Macroeconomics", printed by Colmex. The views expressed here are those of the author and do not necessarily reflect those of Banco de México.

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1 Introduction

In this paper, I want to review a contemporary branch of the informal sector literature that studies the interaction between the informal sector and aggregate variables. From my perspective, this branch is the most promising area of research on informality, because as opposed to previous work, it analyzes the issue from a wider perspective.

The selection of papers surveyed here all analyze the issue of informality building answers from the bottom up. That is, a common strategy throughout is to ask how individual decisions are affected by the presence of informality and, in turn, how these distorted individual decisions add-up and impact on aggregate variables. In order to achieve this goal, the group of papers I discuss below all make use of Dynamic General Equilibrium (DGE) models. The goals of this paper are threefold: 1) to identify the key ingredients in these DGE models; 2) to present the most important modeling alternatives employed in this literature; and 3) to outline the main challenges that this group of authors face.

The traditional literature used to have what La Porta and Shleifer (2008) called “a romantic view” of informality in which the informal sector was that part of the economy where economic activity was being undertaken in the most “pure and efficient” way\(^1\). This view is now challenged by the modern literature via the use of DGE models. Both views start from the same principle: informal firms do not pay taxes or comply with regulation, which in principle makes them more efficient. However, in the DGE models informality introduces distortions on several margins of the economy. For example, occupational and schooling choices, the scale of operation of the firm, and the allocation of resources across plants, among others. This, in turn, affects productivity and output. In this paper, I use a classical general equilibrium model of the informal sector based on Rauch (1991) to illustrate the tradeoff between distortions originating from policies associated with the formal sector, and those associated with the informal sector. As a result of the coexistence of these two types of distortions, a U-shaped relationship between aggregate output and the size of the informal sector exists.

The trade-off above is present in many models of the informal sector. For example, in models with credit frictions, reducing informality on the one hand is good because it softens the credit constraints faced by firms; but, on the other hand is bad because more firms face taxes as the formal sector expands. Similarly, in models with incomplete tax enforcement, reducing informality entails a positive force because it reduces distortions on firm’s scale and on occupational choices; but it also entails a negative force because taxes increase for previously untaxed firms. The exact position and shape of the relationship between output and informality that captures this trade-off depends heavily on the choice of the determinants of the informal sector and on the specification of the way these affect the economy. DGE models provide a framework where all these issues can be addressed.

While early papers focused mainly on the determinants of informality, DGE models not only incorporate the informal sector as an endogenous object, but they also provide a framework where the determinants of informality can be linked to the performance of aggregate variables, such as unemployment and productivity. Rauch (1991) was the first to introduce informality in a general equilibrium model. He, however, was only interested in the determinants of informality and not on the effects that the policies

\(^1\)One important exponent of this view is de Soto (1989)
that generate informality have on aggregates (perhaps this was due to the computational burden of the
time or the tradition of his predecessors). In section 4.1, I will perform a numerical exploration of the
aggregate implications of informality in Rauch’s model.

To the best of my knowledge, the first contemporary article that goes beyond studying the determi-
nants of informality is Antunes and Cavalcanti (2007). These authors introduce a model to analyze how
much of the differences in the size of the informal sector and on income per capita across countries can
be accounted for by differences in the regulation of entry and on the level of enforcement of financial
contracts.

One challenge in this literature is to find a systematic way of identifying the determinants of infor-
mality that are relevant for aggregate outcomes. Motivated by the influential work of de Soto (1989),
the regulation of entry has received substantial attention over the last two decades. Similarly, credit con-
straints seem to be a preferred determinant of informality by many authors (see for example, D’Erasmo
and Moscoso Boedo (2012); Amaral and Quintin (2006, 2010)). One hypothesis is that informal firms
are credit constrained and that improving the institutional framework to allow for a better compliance of
financial contracts will increase credit and reduce informality. Nonetheless, the relevance of these two
determinants has been challenged by important developments in the literature. First, regarding credit
constraints the research of Midrigan and Xu (2010) and Moll (2010) shows that when constrained en-
trepreneurs are allowed to accumulate assets, self-financing can undo the capital misallocation that arises
from financial frictions, and make TFP distortions vanish in the long run. Thus, when we try to focus
on the determinants of informality that are important for long-run aggregate outcomes, credit constraints
seem irrelevant. This is corroborated empirically. Following the discussion in Kaplan et al. (2007), the
evidence suggests that informal firms are not financially constrained. In ENAMIN (2008)\textsuperscript{2} the owners
were asked about the way they financed their start-up costs; 50.5% mentioned that they used their own
savings, and only 1.2% used credit from commercial banks. This may mean that access to credit of small
firms is difficult, or alternatively, that demand for credit is low. However, the owners were also asked
which was the main problem faced by their business; only 2.39% answered that lack of credit was a
problem. Therefore, informal entrepreneurs do not seem to be affected by credit constraints.

Second, empirical studies have found that changes in the regulation of entry have little impact on
informality. In fact, Kaplan et al. (2007) study a reform in Mexico that reduced entry costs in a significant
way, but had almost no impact on informality. The authors suggest that the reason might be that even
though entrepreneurs save entry costs because of the reforms they are also aware that they will have to
face a large tax burden if they decide to enter the formal sector.

In Leal Ordóñez (2013) I concentrate on the tax compliance aspect of informality. I study how
the way firms avoid taxes affects output and productivity. In response to incomplete tax enforcement I
find that firms reduce capital demand, which in turn impacts on capital accumulation. Incomplete tax
enforcement also distorts occupational choices in the economy and misallocate resources across firms.
I find that distortions associated to the informal sector are significant: if Mexico was able to increase
its tax enforcement capacity to a level high enough to eliminate informality entirely, productivity and

\textsuperscript{2}Encuesta Nacional de Micronegocios, INEGI, 2008.
capital accumulation would increase, which will translate into an output increase of around 34% (for a version of the model with monopolistic competition) relative to the output level in the economy with low enforcement and informality.

In conclusion, the use of DGE models that include heterogenous firms has made progress possible on the understanding of informality and its consequences. Looking to the future, more research is needed to understand how informality affects the economy when other sources of heterogeneity are considered (e.g., sectorial and occupational heterogeneity).

In the next section, I continue with a brief history of the concept of informality in order to put into perspective its recent developments. Then, I discuss the main features of a class of models with heterogeneous firms and informality. I do this in the context of the pioneering work of Rauch (1991). In section 4, I analyze the basic distortions induced by informality; and in section 5, I discuss some variants to the basic model of informality. Finally in section 6, I provide some thoughts on the paths that the literature could take in the forthcoming years, as well as discussing the areas of opportunity for future research.

2 A brief history of the informal sector concept

The concept of “the informal sector” has been present in the economics profession for more than 40 years. Its origin is attributed to Hart (1970) (see, e.g., Sethuraman, 1976) and to an International Labor Office (ILO)’s staff report after an “employment mission” to Kenya in 1972 (see, e.g., Bangasser, 2000). The context in which the informality concept was born is important, because it determined the path that the literature would take for the first few decades. For mainstream development economists in the 1950’s and 1960’s, efforts had to be put on capital formation and infrastructure; labor issues were generally of second interest. The ILO’s employment missions were meant to attract the attention of development experts on employment, particularly on the deficit of modern jobs in developing countries and the way this deficit was being filled by informal income opportunities.

The development paradigm back then was grounded on the dualistic view of development (Lewis, 1954) which assumed that a worker could earn different wages depending on the sector he/she was hired; equivalently, it assumed that formal jobs were better than informal jobs, and that informal workers queue for these better jobs. Therefore, “subsistence labor” (i.e. non-modern / informal jobs) were simply a symptom of underdevelopment: as industrialization would take off, workers would be absorbed into the formal sector.

With time, panel data of employment surveys became widely available and an important piece of information was brought into the analysis: there exists substantial work flows into and out of informality. This data shook the foundation of the dualistic view of informality because it provided evidence that many individuals optimally decide to be informal.

As a result of this historical events, a considerable amount of effort within the informal sector literature has been placed on the dualistic/segmented vs. free mobility labor markets debate. The debate

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continues to this day (see Fields (2004) and Perry et al. (2007), for two recent contrasting views). Thus, much of the literature has focused on the optimal decisions of workers and its implications on employment levels and the size of the informal sector. Only recently the focus of researchers has started to move towards the behavior of firms. There is now a renewed interest on informality due to the availability of firm level data. An increasing number of authors are studying the decisions of informal firms, and how institutions and policy affect them. Today, the informal sector is seen as a source of resource misallocation. This has brought a twist in the informal sector literature, as many researchers now recognize the role of the informal sector as a potential cause of underdevelopment.

One important class of models recently used to study informality are those with search and matching frictions (see, e.g., Albrecht et al. (2009), Bosch and Esteban-Pretel (2012), Fugazza and Jacques (2004), Meghir et al. (2012); Esteban-Pretel and Kitao (2013); Zenou (2008)). These models lie in the middle ground of the two views mentioned above, recognizing some role for choice and some role for chance. In these models, formal jobs are “better” than informal ones (offer higher wages on average, are less risky, and are attached to more benefits), but informal workers might still optimally decide to accept an informal job offer because of different reasons. For example, informal job offers might arrive more frequently than formal job offers, or the ability of a worker might not suit the formal sector requirements. This is also a promising area of research that deserves attention, but I would like to focus on a different class of models that emphasize the distortions suffered by firms in the presence of informality.

3 A class of general equilibrium models

I will focus on a class of models in the informal sector literature that includes two main ingredients. The first ingredient is heterogenous firms. Informal firms tend to be small so the use of heterogenous firms in the model is the first step to deliver this equilibrium characteristic. The second ingredient consists of a group of features in the model that define the net-gain of operating informally. In this class of models, the net-gain of informality is decreasing with firm size and thus only small firms have an incentive to choose informality.

This second ingredient is one aspect where the literature takes different approaches. Typically there is a gain enjoyed by informal firms: this gain could be due to tax evasion, avoidance of entry costs, or lack of compliance of minimum wages, among other things. Additionally, there is a cost that increases with firm size. This cost can be, for example, in the form of more stringent tax/regulation enforcement for larger firms or in the form of allowing formal credit only for larger firms. This is why the net gain (gain minus cost) is decreasing with firm size.

These two ingredients are closely related to those used in the macro literature that studies idiosyncratic distortions across heterogenous plants. In this literature, identified in the work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2007), while firms face policy distortions that are more general than the distortions modeled by the informal sector literature, they certainly share the same spirit. Therefore, in section 4, I will present some advances from this macro literature that complement the findings of the informal sector literature.
I would like to start with the first formal contribution in the informal sector literature associated with Rauch (1991). This model was influenced by the wave of evidence showing reverse mobility, i.e., workers moving from formality to informality. He was at the same time, not very inclined to abandon the dualistic view of labor markets. Thus, Rauch developed a model where even by assuming free mobility across sectors a sort of segmentation occurs in the labor market. Here I present a simplified version of Rauch’s model.

3.1 The distortion-free economy

The environment departs from Lucas (1978)’s occupational choice model. There is a continuum of individuals with ability \( z \in [0, \bar{z}] \), and distributed according to cdf \( F(z) \). Consider first the simplest version of the model without government and informality, that is, without distortions. The individuals can choose to operate either as entrepreneurs, or as employees. If an individual decides to be an employee (s)he receives the wage \( w \); on the other hand, if the individual chooses to be an entrepreneur, (s)he has access to a production function with decreasing returns to scale: \( y(z) = z l^\theta \), with \( 0 < \theta < 1 \). Now consider the problem of an entrepreneur with ability \( z \), given \( w \):

\[
\pi(z, w) = \max_l \left\{ z l^\theta - w l \right\}.
\]

For later reference, let \( l(z, w) \) be the optimal labor demand for individual \( z \) and \( y(z, w) \) be optimal output. Notice that \( \pi(z, w) \) is increasing in \( z \) (an application of the envelope theorem), while \( w \) is just a constant. Not surprisingly, for any \( w > 0 \) there exists a threshold \( \hat{z} \), such that \( \pi(\hat{z}, w) = w \), to the left of which all individuals are better-off being employees (i.e., \( w > \pi \)), and to the right of which individuals are better-off being entrepreneurs (i.e., \( w < \pi \)). Figure 3.1 shows how \( \hat{z} \) is determined.

A competitive equilibrium for this economy consists on a threshold \( \hat{z} \), wages \( w^* \), and quantities \( l(z, w^*), \forall z \geq \hat{z} \), such that:

1. Each entrepreneur solves his/her problem;

2. \( \pi(\hat{z}, w^*) = w^* \);
3. Labor market clears: \( \int_\hat{z}^\infty l(z, w^*) dF(z) = F(\hat{z}) \).

By standard arguments this equilibrium exists and is unique. Lucas (1978) shows that the equilibrium allocation of this undistorted economy is also the solution to a social planner’s problem that maximizes aggregate output.

In terms of the equilibrium characterization, it can be shown that the equilibrium threshold is given by:

\[
\hat{z} = \left[ \frac{\theta}{1 - \theta} \frac{\int_\hat{z}^\infty z^{\frac{1}{1-\theta}} g(z) d\hat{z}}{G(\hat{z})} \right]^{1-\theta},
\]

(3.1)

which is one equation with one unknown. The parameter \( \theta \) measures the returns to scale at the firm level, which is important in determining the threshold \( \hat{z} \). Thus, the returns to scale parameter is crucial to understand how resources are allocated across establishments in equilibrium. To see this, consider the problem of a social planner in the extreme case when \( \theta = 1 \). In this case all resources are allocated to the most productive establishment and \( \hat{z} = z_{\text{max}} \). As \( \theta \) moves away from one, decreasing returns become important, and allocating some resources to other firms becomes efficient.

Finally, notice that aggregate variables are given by the sum of individual decisions. For example, aggregate output is given by \( Y = \int_\hat{z}^\infty y(z, w^*) dF(z) \), the mass of entrepreneurs in equilibrium is given by \( \text{EntrMass} = 1 - F(\hat{z}) \), and the average entrepreneurial ability is: \( \text{AvEntrAb} = \left( \int_\hat{z}^\infty z^{\frac{1}{1-\theta}} dF(z) \right) / (1 - F(\hat{z})) \).

### 3.2 A minimum wage economy with full enforcement

As a second step, Rauch (1991) considers an economy with perfect enforcement of minimum wages. Notice that this policy is naturally distortionary: since the government does not have any other role in the model, minimum wages simply introduce a deadweight loss. The author argues that in this economy there is no equilibrium, and that labor supply will exceed labor demand when \( \bar{w} > w^* \). Rauch (1991) concludes that in this economy unemployment will arise.

The problem of entrepreneur \( z \) is as follow:

\[
\pi_{FE}(z, \bar{w}) = \max_l \left\{ zl^\theta - \bar{w}l \right\}.
\]

For future reference let \( l_{FE}(z, \bar{w}) \) denote the optimal labor choice of entrepreneurs under full enforcement. The threshold \( \hat{z}_{FE} \) is defined in a similar way as \( \hat{z} \) in the previous section, except that the equilibrium wage in condition 2 above is replaced by the minimum wage: \( \pi(\hat{z}_{FE}, \bar{w}) = \bar{w} \). Notice also that condition 3 above is not required. With this, we can calculate aggregate variables. However, these are not the result of an equilibrium outcome as in the case of the undistorted economy.

**Distortions induced by formality.** The introduction of a minimum wage increases the cost of labor to all entrepreneurs, thereby affecting production decisions. As explained above, under full enforcement there will be an excess supply of labor, which makes it difficult to compare the undistorted with the
distorted outcomes. Nonetheless, by inspection of Figure 3.1, it is clear that $\hat{z}_{FE} > \hat{z}$. The reason for this is that with the introduction of the minimum wage, the horizontal straight line shifts up, while the curved line shifts down and to the right. Thus, aggregate production can not be efficient under these circumstances because a group of entrepreneurs is missing, and less labor is being used in production as a result of the increase in labor costs ($l_{FE}(\hat{z}, \bar{w}) < l(z, w^*)$, $\forall z \in [\hat{z}_{FE}, z_{max}]$).

### 3.3 A minimum wage economy with imperfect enforcement

The next step consists of introducing informality into the model. Rauch (1991) uses a cost of informality that varies with firm size, which has now become standard in the literature\(^4\). In his model, the minimum wage ($\bar{w}$) is only enforced on firms larger than certain size $\bar{l}$. This can alternatively be rationalized using a probability of detection equal to 0 if labor is less than or equal to $\bar{l}$, and equal to 1, otherwise (see, e.g., de Paula and Scheinkman, 2007 and Bigio and Zilberman, 2011). Since more informality implies a lower compliance with minimum wages, the economy tends to move towards efficiency as informality increases. However, informality introduces other type of distortions as well, which will be analyzed in section 4. In this section I will focus on describing the way informality is generated in Rauch’s model.

Firms that do not comply with minimum wage requirements are labeled “informal”; while firms that comply with such regulation are called formal. Employees working in informal firms are labeled “informal employees”, and the formal employees are defined in an analogous way. Note that there is never going to be a case that an individual chooses to operate as an informal entrepreneur and at the same time chooses labor greater than $\bar{l}$, otherwise the entrepreneur will be caught and punished. Therefore, informal firms enjoy of lower labor costs $w < \bar{w}$, but they also suffer a constraint on the amount of labor they can hire $l \leq \bar{l}$. Notice that this implies that the net gain of being informal varies with firm size. Firms with low $z$, which naturally tend to operate at a low scale, enjoy the reduced labor costs but do not bear any constraints: thus they are implicitly subsidized.\(^5\) Firms with medium levels of ability find it attractive to distort the amount of labor hired in order to remain undetected by the government and still enjoy the benefit of paying low wages. The largest firms are better-off complying with minimum wages because their marginal productivity of labor is high and reducing labor to $\bar{l}$ signifies an important production loss that does not compensate the gain of lower labor costs.

While labor is homogenous in this model, the existence of limited minimum wages enforcement allows identical employees to earn different wages. This is the spirit of the dualistic view of labor markets. Rauch (1991) assumes that employees are randomly assigned to each sector, so not every employee earns the formal sector wage $\bar{w}$, and unlucky individuals in the informal sector earn less than the minimum wage. This idea can be simplified by introducing stages in the timing of the decisions taken by agents in the model (this is implicit in Rauch’s analysis).

Assume there are two stages and that individuals have to take their occupational choices in stage one and production decisions in stage two. Also, in stage 2 employees are sorted into the formal and informal

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\(^4\)It was already a well-established empirical fact that informal firms are small.

\(^5\)As in Rauch (1991), I assume that the minimum wage is binding. This assumption is one important critique to Rauch’s paper, because in many developing countries with large informal sectors, minimum wages do not bind.
sectors randomly. Given occupational choices of stage 1, there is no possibility to revert such decisions in the next stage. Furthermore, individuals make occupational choices facing uncertainty regarding the sector (and the wage) they will get if they choose to be employees. Nonetheless in stage 1 individuals have some information at hand: they are aware of the market (informal) wage, of the minimum wage, and of the probability of getting a formal sector job. Therefore agents in this model are faced with the following occupational discrete choice: a) be a formal entrepreneur; b) be an informal entrepreneur; or c) be an employee.

If an individual \( z \) decides to be a formal entrepreneur (s)he faces the minimum wage and solves:

\[
\pi_F(z, \bar{w}) = \max_l \left\{ zl^{\theta} - \bar{w}l \right\}.
\]

Let \( l_F(z, \bar{w}) \) denote the optimal labor choice of formal entrepreneurs. Instead, if an individual decides to be an informal entrepreneur, the problem is:

\[
\pi_I(z, w) = \max_l \left\{ zl^{\theta} - wl \right\} \quad s.t. \quad l \leq \bar{l},
\]

and similarly, let \( l_I(z, w) \) denote the optimal labor choice of formal entrepreneurs. Finally, if the individual decides to be an employee the agent faces a lottery: with probability \( p \) the employee is assigned in the second stage to the formal sector earning \( \bar{w} \), and with probability \( 1 - p \), goes to the informal sector earning \( w < \bar{w} \). Thus, ex-ante employee earnings are given by \( Ew = pw + (1 - p)w \). Notice also that the probability of obtaining a formal sector job as an employee is an endogenous object equal to the amount of labor demanded by formal firms, divided by the amount of people that is willing to become employee:

\[
p = \frac{\int_{z_1}^{z_2} l_F(z, \bar{w}) dF(z)}{F(z_1)}.
\]

Occupational choices in the distorted economy are summarized in Figure 3.2. Entrepreneurial profits are increasing in ability and employee earnings are flat, as in the distortion-free case. Therefore, there exists a threshold \( z_1 \) to the left of which individuals are (ex-ante) better-off being employees. To the right of \( z_1 \) individuals find it more attractive to become entrepreneurs. A second feature of Figure 3.2, is that the profits of informal entrepreneurs (the \( \pi_I(z, w) \) line) are higher than profits of formal entrepreneurs for low levels of ability. This is reverted for high ability levels. The reason for this is that the constraint suffered by informal entrepreneurs becomes more costly for individuals with a large \( z \). Given a labor level, the marginal productivity of labor is higher, the larger is \( z \). Thus, restricting labor to \( \bar{l} \) results in a larger production loss for firms with high \( z \). This trade-off is summarized by threshold \( z_2 \), to the left of which entrepreneurs find it more attractive to become informal, and to the right of which all of them prefer to be formal.

4 Distortions induced by informality

This simple static model is rich enough to deliver interesting distortions induced by informality. There are three type of distortions: 1) on the optimal labor demand of informal firms; 2) on occupational choices; and 3) on the allocation of resources across firms.
The first distortion occurs within firms. It affects a group of informal firms to the left of the threshold $z_2$. Notice that for these firms there is an incentive to remain small in order to avoid detection. They find it optimal to operate with labor equal to $\bar{l}$ and enjoy low costs. Not every informal entrepreneur is necessarily in this situation. In particular, firms with ability just right of $z_1$ might not need to distort their labor choices since they tend to operate at a low scale. This point is illustrated in Figure 4.1. Notice that between $z_1$ and $z_c$ informal firms are unconstrained, while those individuals with ability between $z_c$ and $z_2$ operate with labor exactly equal to $\bar{l}$. Formally, threshold $z_c$ is defined as the one that satisfies: $l(z_c; w) = \bar{l}$.

The second distortion relates to occupational choices. The limited enforcement of minimum wages makes it attractive for a group of individuals to become entrepreneurs instead of employees. This pushes the employee-entrepreneur threshold $z_1$ to the left, thereby distorting occupational choices. This point is illustrated by comparing Figures 3.1 and 3.2.

The third distortion is more subtle. It refers to the way existing resources are allocated across establishments, and it is intimately linked to the type of distortions found in the macroeconomic literature on resource misallocation across heterogenous plants, previously mentioned in the introduction. The main
idea here is that due to limited enforcement of minimum wages, firms with low productivity are allocated more resources than they should when compared to an efficient environment.

4.1 A numerical example

To illustrate the mechanics in the model consider the following numerical example. I use a truncated Pareto distribution for the entrepreneurial ability and assume that it is distributed according to:

$$G(z) = \frac{1 - (\frac{z_{\min}}{z})^s}{1 - (\frac{z_{\min}}{z_{\max}})^s},$$

where $s > 0$ is called the shape parameter and $z \in [z_{\min}, z_{\max}]$. For this particular numerical exercise I need to provide a value for the parameters for the distribution of the entrepreneurial ability $(s, z_{\min}, z_{\max})$, the technology parameter $(\theta)$, and policy parameters: $(\bar{w}, \tilde{l})$. I use the following values: $s = 4.25$, $z_{\min} = 1$, $z_{\max} = 13.05$, and $\theta = 0.66$, and $\bar{w} = 0.9$. With these parameter values, I perform two exercises where I change the quality of tax enforcement. First, I compute the equilibrium when $\bar{l} \to \infty$. In this equilibrium, no firm complies with the minimum wage legislation. Equivalently, this economy is operating without distortions. Second, I compute the equilibrium when $\bar{l} > 0$ is finite. In this economy, the informal sector is large and the distortions discussed earlier are present.

Table 1 presents the results of this exercise. When $\bar{l} = 1000$, there is no entrepreneur that complies with minimum wages (i.e. all are informal). Thus, this economy operates without distortions. When $\bar{l} = 20$, the largest firms become formal. Even though the size of the informal sector is still very large, the distortion induced by minimum wages is present and aggregate output is below the efficient level (98%). Something similar occurs when $\bar{l} = 5$. In this case the share of the formal sector is even larger and more of them are therefore forced to pay higher wages, which in turn distorts aggregate outcomes (output is reduced to 95% of the undistorted level).

Notice that both wages and the average entrepreneurial ability decrease, as $\bar{l}$ decreases. This is consistent with the employee/entrepreneur threshold $z_1$ moving to the left of $\hat{z}$. As $z_1$ moves to the left, more individuals become entrepreneurs and less employees (see the last two rows of Table 1). Consequently, average entrepreneurial ability goes down because the type of new entrepreneurs that enter in operation are the least productive. The mean size of establishments is also smaller, since there are less employees per entrepreneur. This result might be puzzling when compared to the result of full enforcement. In the section above we showed that threshold $\hat{z}_{FE}$ lies to the right of $\hat{z}$. The reason for this is that $\hat{z}_{FE}$ is not an equilibrium outcome, and it only has to be consistent with condition 2 above.

The reader might be tempted to conclude from Table 1, where output increases as informality increases, that the distortions associated with informality are small, and that the distortions that really matter in a quantitative sense are those associated with policies enforced in the formal sector. However, it must be stressed that this numerical exercise is not a calibration and thus it is only used as illustrative purposes. Furthermore, Rauch’s model includes only one feature of informality (minimum wages) and abstracted form taxation, financial constraints, and other features that the current literature has considered important.
4.2 Two sources of distortions: formality and informality

In Leal Ordóñez (2013), I use a model calibrated for Mexico to show that both distortions, those associated with the formal sector policies, and those that arise due to the presence of informality, are important. In fact, when the level of enforcement is plotted against equilibrium output, a U-shaped curve arises (see Figure 4.2). This means that for high levels of informality, output is high and close to the level of the undistorted economy\textsuperscript{6}. However, for medium levels of informality, output is low because both distortions from the formal and informal sectors are large. When informality is very low, output is again high because there is only distortions associated to formality.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{informality_output.png}
\caption{Informality and output}
\end{figure}

\textsuperscript{6}This conclusion is obviously overturned once we allow for a less inconsequential role of government expenditures in the model. For didactic purposes, I have assumed that all revenue is rebated to the households through a lump-sum transfer. See Loayza (1996), for a model where the tax revenue is used to provide public infrastructure.

\begin{table}[h]
\centering
\caption{Distortions induced by informality (numerical exercise)}
\begin{tabular}{|c|c|c|c|c|}
\hline
 & $l = 1000$ & $l = 20$ & $l = 5$ & $l = 2.5$ \\
\hline
Y & 1 & 0.98 & 0.95 & 0.91 \\
Share of informal employees & 1 & 0.97 & 0.93 & 0.89 \\
w & 1 & 0.96 & 0.90 & 0.84 \\
Mean size & 5.91 & 4.93 & 3.77 & 2.72 \\
Average entrepr. ability & 1 & 0.96 & 0.92 & 0.86 \\
Mass of Employees & 1 & 0.97 & 0.92 & 0.85 \\
Mass of Entrepreneurs & 1 & 1.16 & 1.45 & 1.86 \\
\hline
\end{tabular}
\end{table}
4.3 Idiosyncratic distortions across heterogenous plants

One particular distortion associated with informality that has been the study of recent papers identifies informality as a source of resource misallocation. This view associates informality with idiosyncratic distortions across plants. In order to see how this can operate I quickly revise an important paper on resource misallocation and heterogenous plants Restuccia and Rogerson (R&R, 2008). The goal of R&R is to analyze plant idiosyncratic distortions within the context of a standard neoclassical model. They depart from a simplified version of the model of firms dynamics found in Hopenhayn and Rogerson (1993).

In the R&R model, there is a representative household that lives infinitely, who does not value leisure, and has standard preferences over consumption:

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

where $C_t$ is consumption in period $t$ and $\beta$ is the discount factor. Endowments consists on $K_0$ units of capital at date 0, and one unit of time each period.

Next I briefly describe the technology. Although it is more elaborated than the previous static model, both are designed to deliver a non-degenerated distribution of resources across heterogenous plants in equilibrium. While Lucas (1978)'s model does this using occupational choices, Hopenhayn and Rogerson (1993) develop a model of firms dynamics. Each plant has a production function $zf(k, l)$, where $z$ is plant productivity, $l$ is labor services, and $k$ is capital. The function $f$ exhibits decreasing returns to scale. Plant productivity is constant over time, but each period plants face an exogenous probability of death equal to $\lambda$. Exit realizations are iid across plants and across time. There is also an entry cost $c_e$ that must be paid in order to create a new plant. After this cost is paid, plants take a productivity draw $z$ form a distribution with cdf $H(z)$.

The problem of an incumbent plant is as follows:

$$\pi(z, \tau) = \max_{k, l} \left\{ (1 - \tau)zk\theta_k l\theta_l - wl - rk - cf \right\},$$

where $\tau$ is an idiosyncratic tax/subsidy that the plant bears and $c_f$ is a fixed cost of production. Optimal input choices are given by:

$$k(z, \tau) = \left( \frac{\theta_k}{r} \right)^{\frac{1-\theta_l}{1-\theta_k}} \left( \frac{\theta_l}{w} \right)^{\frac{\theta_l}{1-\theta_k}} \left( z(1 - \tau) \right)^{\frac{1}{1-\theta_k}};$$

$$l(z, \tau) = \left( \frac{\theta_k}{r} \right)^{\frac{1-\theta_l}{1-\theta_k}} \left( \frac{\theta_l}{w} \right)^{\frac{\theta_l}{1-\theta_k}} \left( z(1 - \tau) \right)^{\frac{1}{1-\theta_l}}.$$

In order to see how idiosyncratic distortions misallocate resources, I will focus on one property of the equilibrium efficient allocation and describe the way this is affected by idiosyncratic distortions. For this, I take two plants $z_i$ and $z_j$, such that $z_i < z_j$, and compare their corresponding optimal input choices.
In the absence of distortions, relative labor depends only to the relative productivities of these plants:

\[
\frac{l(z_i, 0)}{l(z_j, 0)} = \left( \frac{z_i}{z_j} \right)^{\frac{1}{1-\theta_l-\theta_k}}.
\]

In particular, this implies that in the absence of distortions and since \( z_i < z_j \), plant \( z_i \) is smaller (in terms of labor) than plant \( z_j \). Since \( z_i \) and \( z_j \) are generic, this holds for any two plants in operation. This means that the distribution of labor is intimately linked to the distribution of \( z \) in equilibrium. However, when distortions differ across plants (\( \tau_i \neq \tau_j \)) the relative labor is now:

\[
\frac{l(z_i, \tau_i)}{l(z_j, \tau_j)} = \left( \frac{z_i(1 - \tau_i)}{z_j(1 - \tau_j)} \right)^{\frac{1}{1-\theta_l-\theta_k}},
\]

which implies that the allocation of labor across plants is affected by the distribution of distortions \( \tau \) across plants. To see this clearly consider the following example. Let \( z_i = 2, \ z_j = 1, \) and \( \theta_l + \theta_k = 0.8 \). Thus, the relative labor of these two plants in the absence of distortions is \( \frac{l(z_i, 0)}{l(z_j, 0)} = 0.42 \). Now let one plant being subsidized and the other taxed. Specifically, let \( \tau_i = -0.5, \) and \( \tau_j = 0.5 \). Relative labor becomes: \( \frac{l(z_i, \tau_i)}{l(z_j, \tau_j)} = (3 * 0.5)^{1.25} = 1.66 \). This shows that idiosyncratic distortions can potentially distort resource allocation across plants in an important amount. In this example, a plant that is half as productive as other ends up being 66% larger instead of being 60% smaller, as the efficient allocation dictates.

R&R calibrate their model to the U.S., and show that when distortions are correlated (e.g., subsidizing the least productive plants and taxing the most productive ones) total factor productivity and output can be reduced by up to 30%.

In conclusion, informality is a source of resource misallocation because it implicitly subsidizes low productive firms, while it taxes high productive ones. In addition to this, and as mentioned before, there are other distortions associated to informality. For instance, occupational choices can also be distorted, as well as the scale of operation of firms.

5 Variants of models with a discrete occupational choice

There are four key characteristics of Figure 3.2 that define how occupational choices are made. First, employee earnings are constant across ability levels, while entrepreneurial profits are increasing. Second, notice that for low ability levels the slope of the curve of informal entrepreneurial profits is larger than the slope of formal entrepreneurial profits. Third, as \( z \) grows, the slope of the red line hits \( \bar{\theta} \) and stays constant. Fourth, the slope of the curve of entrepreneurial formal profits keeps growing as \( z \) increases and eventually formal profits become larger than informal profits. These key features of Figure 3.2 (with some variants) are present in a number of general equilibrium models that focus on the informal sector (see, e.g., de Paula and Scheinkman (2007), Prado (2011), Quintin (2008), Fortin et al. (1997), Choi and Thum (2005), and Gollin (1995), among others).
5.1 Own-account workers

One of the variants the literature has considered is the inclusion of one more alternative occupation: *own-account workers*, or *one-person firms* (see Gollin (1995, 2002)). The motivation of these papers stems from the fact that the labor force in developing countries is composed of a large fraction of this type of workers. The strategy to obtain a positive mass of own-account workers in equilibrium is similar to the one followed when informal firms are allowed. Typically, own-account workers cannot hire employees and only have access to the following production function: \( y = Az^\theta \), where \( z \) is entrepreneurial ability as before, \( l \leq 1 \) is labor, and \( A > 1 \) is a scale factor that captures productivity differences specific to the own-account workers. Again, what is important is the slope of the own-account profits curve. For low levels of \( z \), this slope is larger than the slope of the curve of formal entrepreneurial profits because \( A > 1 \). However, as \( z \) increases the slope hits \( A \) and becomes constant. Thus, eventually as ability increases, employer’s profits become larger than own-account profits.

5.2 Capital accumulation

One more variant in the literature are models with capital accumulation (see, e.g., de Paula and Scheinkman, 2007). These models explore how informality affects not only occupational choices and resource allocation in the economy, but also investment decisions. I next describe the model used in Leal Ordóñez (2013).

There is a representative household populated by a continuum of individuals of mass 1 as in Guner et al. (2008). This household takes consumption decisions, accumulates capital, and makes occupational choices for each member. As in Rauch (1991), members are endowed with entrepreneurial ability \( z \) distributed in the population according to cdf \( G(z) \). Each member can have one out of three alternative occupations: formal entrepreneur, informal entrepreneur, or employee. As in Rauch’s paper the net gain of becoming informal entrepreneur is decreasing with size. In this case, formal firms face an exogenous cost in the form of an output tax, where it is assumed that firms with capital larger than \( b \) face a probability of detection equal to 1, and zero otherwise. Thus, occupational choices are described by a picture similar to Figure 3.2.

The household has standard preferences:

\[
\sum_{t=0}^{\infty} \beta^t u(C_t),
\]

where \( C_t \) is consumption in period \( t \) and \( \beta \) is the discount factor. Endowments consists on \( K_0 \) units of capital at date 0, and one unit of time each period for each member. As is standard, capital accumulates according to \( K_{t+1} = X_t + (1 - \delta)K_t \). The household chooses sequences of consumption, capital, and each member’s occupation, taking as given the prices, taxes \( \tau_v \), a lump sum transfer \( T_t \), and the constraint on capital of informal firms, given parameter \( b \) to maximize lifetime utility. The problem is:

\[
\max_{\{C_t, K_t, I_t(z), F_t(z)\}} \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t) \right\}, \quad (5.1)
\]
subject to the following budget constraint:

\[ C_t(z) + K_{t+1} - (1 - \delta)K_t = r_tK_t + E(w_t, r_t; \tau_y, b) + T_t, \forall t \]  

(5.2)

where \( K_0 \) is given, \( E(w_t, r_t; \tau_y, b) \) is a function that describes earnings of each household, and \( I(z; w, r) \) and \( F(z; w, r) \) are index functions that represent occupational optimal decisions.

Dynamic models are useful to understand the relationship between informality and capital accumulation. In Leal Ordóñez (2013), the probability of being detected depends on capital and therefore capital decisions of informal firms are affected. In particular, there is a group of firms that find it optimal to operate with capital exactly equal to \( b \) in order to remain undetected by the government and still enjoy tax evasion. This feature of the equilibrium affects the aggregate demand of capital and hence capital accumulation. In a calibrated version of the model with perfect competition, I found that Mexico’s output would be 17% higher if tax enforcement was improved to completely eliminate informality. A big fraction of this effect is explained by an increase in capital accumulation of 45%.

5.3 Credit constraints

Amaral and Quintin (2006) build a dynamic model with credit constraints and occupational choices. As before, individuals can choose to be employees, formal entrepreneurs, or informal entrepreneurs.\(^7\) In the formal sector profits are taxed at a rate \( \tau \), while informal managers do not pay taxes. Thus, as opposed to the model of Rauch (1991), the formal/informal tradeoff does not rely on the way taxes are enforced, but rather on the presence of financial frictions. The marginal productivity of capital is higher for individuals with high ability (ceteris paribus) and therefore they tend to operate with large amounts of capital. This makes the formal sector, where credit is available, more attractive for large firms.

The credit market works as follow. Entrepreneurs can self-finance part of their operating capital with their own savings/wealth. They can also borrow from a financial intermediary that charges an interest rate \( r \). These loans occur within the period. There exists limited enforcement of financial contracts, and borrowers can choose to default on their debts. This is another aspect where formal and informal managers differ. If a formal entrepreneur chooses to default they are subject to a penalty that represents a fraction \( \phi \) of their income. This parameter captures the strength of contract enforcement in the economy. In contrast, it is assumed that informal firms are not subject to this penalty. Notice that this creates incentives for informal entrepreneurs to default. Since debt contracts must be self-enforcing, this also means that informal entrepreneurs will not have access to credit in equilibrium.

To illustrate the mechanics of the model, I revise the entrepreneur’s problem in Amaral and Quintin (2006). We can separate the problem in two stages. First, we compute profits for any level of capital that the entrepreneur can operate with. Second, the individual chooses the amount of capital that he is going to self-finance, and the amount he is going to borrow. Consider first the problem for individual \( z \) with given capital \( k \) (before taxes):

\(^7\)See also Antunes and Cavalcanti (2007).
\[ \pi(z, k) = \max_l \left\{ zk^\theta_l l^\theta_l -wl - k(1 + r) \right\}, \]

where for later I define \( k^*(z) = \arg\max_k \pi(z, k). \)

Now let \( a \) be the value of the assets that the individual possess. Let \( s \) be the amount of capital that is self-financed, and let \( d \) be the amount that is being borrowed from the financial intermediary. The income of a formal entrepreneur with ability \( z \) and wealth \( a \) is given by:

\[ V_F(z, a) = \max_{0 \leq s \leq a, d \geq 0} (1 - \tau)\pi(z, s + d), \]

subject to

\[
(1 - \tau)\pi(z, s + d) + a(1 + r) \\
\geq (1 - \phi)(1 - \tau) [\pi(z, s + d; w) + (s + d)(1 + r)] + (a - s)(1 + r).\]

This last restriction is the incentive compatibility constraint that states that the value of repaying the loan has to be higher than, or equal to, the value of defaulting. In other words, the bank will only lend an amount \( d \geq 0 \) such that defaulting is not attractive.

Amaral and Quintin (2006) show that in equilibrium, the amount that formal entrepreneurs can borrow rises with their own savings and with their ability. Both results are intuitive. Given \( z \), as savings increase, so does self-financing, which in turn, rises the opportunity cost of defaulting. Similarly, given \( a \), as entrepreneurial ability increases, the incentive compatibility constraint becomes less binding which increases the opportunity cost of default. The paper also shows that, given ability \( z \), wealthier entrepreneurs are more likely to opt for the informal sector. The reason for this is that entrepreneurs can only borrow in the formal sector. Poor entrepreneurs prefer the formal sector, despite the taxes, because this way they can borrow and increase capital. However, rich entrepreneurs are able to self-finance and thus they prefer the informal sector, since \( \tau > 0 \). Furthermore, Amaral and Quintin, 2006 show that informal entrepreneurs operate with less capital than formal entrepreneurs.

In their model, Antunes and Cavalcanti (2007) additionally include entry costs into the formal sector. They calibrate the model to the U.S. and perform a number of counterfactual exercises. For example, they show that when the parameter that controls the quality of enforcement of financial contracts decreases by a factor of 2 from the baseline economy, the size of the informal sector increases 19% and measured output decreases by 33%. In contrast, they show that endogenous variables are less responsive to changes in entry costs. For example, when the entry cost increases by a factor of 8, the informal sector size increases by only 3.5% and the official output is only 95% of its original level.

### 5.4 Heterogenous labor services

Amaral and Quintin (2006) use a production function at the plant level that exhibits decreasing returns to scale and capital-skill complementarities. An entrepreneur with ability \( z \) operates the following produc-
tion function: \( z_F(k, l_u, l_s) = z[\min(k, l_s)]^{\alpha} l_u^\theta \), where \( 0 < \alpha + \theta < 1 \), \( k \) is capital, \( l_u \) is unskilled labor, and \( l_s \) is skilled labor. Since credit is only available for formal firms, they tend to operate with more capital than informal firms in equilibrium. This also implies that formal firms rely more intensively on skilled labor because unskilled labor is a better substitute for capital than skilled labor (i.e. there is capital-skill complementarities).

Unfortunately Amaral and Quintin (2006) did not included a quantitative assessment of the implications of their model. However, D’Erasmo and Moscoso Boedo (2012) construct a model along the same lines. They calibrate the model to the U.S. and perform a number of counterfactuals where they analyze joint changes in the costs of formality. They find a complementary effect of entry costs and financial frictions when changing the formality/informality tradeoff from the one in the U.S. to those in developing countries. Relative to the U.S., their model generates up to a 37% decrease in TFP, 10 times larger informal sectors and a 60% decline in the stock of skilled workers when moving to parameters that characterize developing countries.

6 Conclusion

This paper has presented a review of articles that belong to a class of models in the contemporary informal sector literature. This literature is closely related to a macro literature that studies the role of idiosyncratic distortions across plants. In these models it is clear that the informal sector plays a distortionary role in the economy, which challenges the romantic view of the informal sector as receptor of the most pure and efficient kind of economic activity, and as an engine for development. This literature, emphasized distortions associated with the presence of informality not only within firms, but across firms. It includes two key ingredients: heterogenous firms and a net gain of informality that decreases with firm size. In this class of models, informality distorts the scale of firms and the allocation of resources across firms.

Additionally, the literature has explored distortions on other margins such as occupational choices, and physical and human capital accumulation. When the results of all these papers are put together, it is clear that the informal sector is not only a symptom of underdevelopment, but is also a cause of underdevelopment. There is, nonetheless, a tradeoff when reducing informality: more taxes vs. less distortions.

In recent years the focus of the macro literature on misallocation across plants has been on the role played by complementarities and links among the several sectors in the economy. To the extent that the informal sector is concentrated on sectors that are linked to the rest of the economy in specific ways, this is an interesting area of research (see, e.g., Jones (2013), and Jones, 2011).

Another area of opportunity is the one related to the role of financial constraints on informality. The macro literature has also had recent important developments in this area. In particular the papers of Midrigan and Xu (2014) and Moll (2010) rise doubts about how relevant financial constraints are for resource misallocation.
References


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